

#### Lecture 16

#### More problems on the Greedy Method

CS 161 Design and Analysis of Algorithms
Ioannis Panageas

## Greedy method

The greedy method is a general algorithm design technique, in which given:

- configurations: different choices we need to make
- objective function: a score assigned to all configurations,
   which we want to either maximize or minimize

We should make choices greedily: We can find a globallyoptimal solution by a series of local improvements from a starting configuration.

Problem 1: Given: a set T of n tasks, each having a start time  $s_i$  and a finish time  $f_i$  (where  $s_i < f_i$ )

Goal: Perform all the tasks using a minimum number of machines. A machine can serve one task at a given time.

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Idea: Sort tasks in increasing order of their **start** time. Assign first task to machine 1 and set K = 1.

When considering a new task, if all machines are busy, create a new machine, set K = K + 1 and assign the new task to the new machine otherwise assign the new task to an available machine.

Problem 2: Given: a set T of n tasks, each having a start time  $s_i$  and a finish time  $f_i$  (where  $s_i < f_i$ )

Goal: Perform as many tasks as possible using one machine. In other words, find the maximum number of non-overlapping intervals.

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Idea: Sort tasks in increasing order of their **finish** time. Perform first task and remove all overlapping tasks with first task. Repeat the same process to the remaining tasks.

Example: 7 Tasks, [1,4], [1,3], [2,5], [3,7], [4,7], [6,9], [7,8]

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Problem 3: You are given a set T of n tasks, each having a deadline time  $f_i$  and profit  $p_i$  if completed and needs one unit of time to be completed. You have only one machine.

Goal: Complete non-overlapping tasks to maximize your profit.

Example: 7 Tasks, (5\\$, 3), (2\\$, 2), (2\\$, 2), (1\\$, 2), (4\\$, 4), (2\\$, 4), (1\\$, 4)

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Solution: Task 2, Task 3, Task 1, Task 5 with profit 13\$

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Idea: Sort tasks in decreasing order of their **profit**. Repeat the following until run out of tasks: Choose first task and schedule it at the latest time possible without exceeding deadline. If not possible, discard the task.

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$$(4\$, 4), (2\$, 2), (2\$, 2), (2\$, 4), (1\$, 2), (1\$, 4)$$

Schedule task (5\$,3) between times 2-3.

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$$(2\$, 2), (2\$, 2), (2\$, 4), (1\$, 2), (1\$, 4)$$

Schedule task (5\$,3) between times 2-3. Schedule task (4\$,4) between times 3-4.

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$$(2\$, 2), (2\$, 4), (1\$, 2), (1\$, 4)$$

Schedule task (5\$,3) between times 2-3. Schedule task (4\$,4) between times 3-4. Schedule task (2\$,2) between times 1-2.

Design and Analysis of Algorithms

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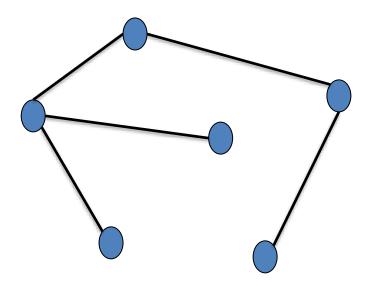
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$$(2\$, 4), (1\$, 2), (1\$, 4)$$

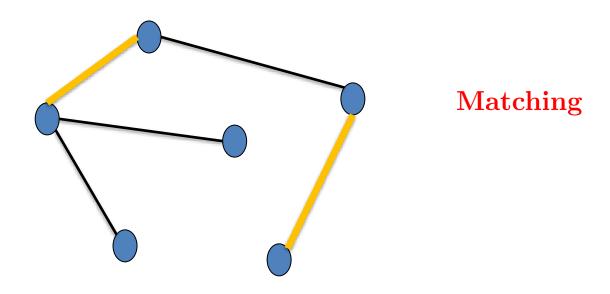
Schedule task (5\$,3) between times 2-3. Schedule task (4\$,4) between times 3-4. Schedule task (2\$,2) between times 1-2. Schedule task (2\$,2) between times 0-1.

Design and Analysis of Algorithms

Definition: Given a graph G, a **matching** is a collection of edges that do not share a vertex.

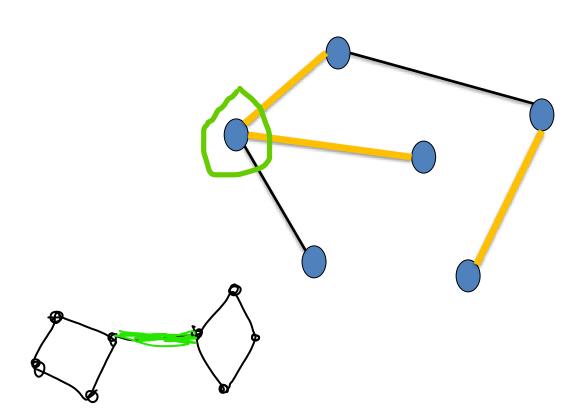


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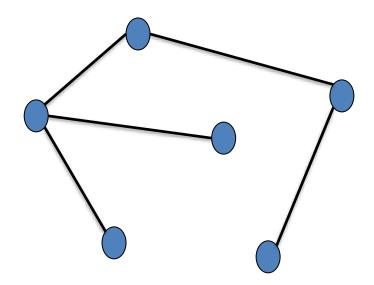
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Not a Matching

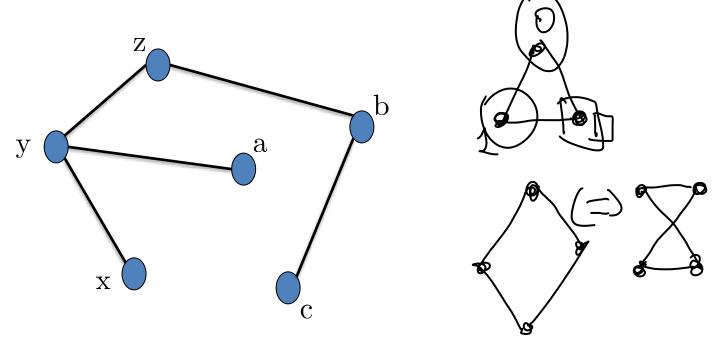
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Problem: Given a tree graph, compute/find a maximum matching.

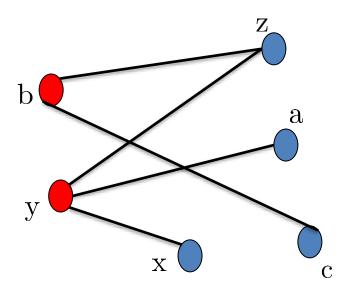
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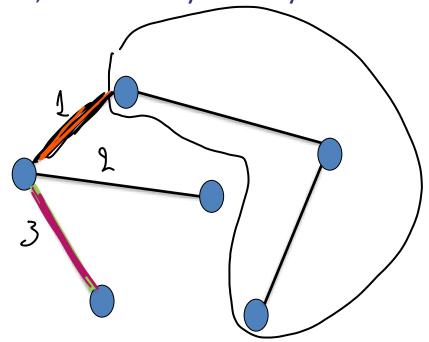


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Trees are bipartite!

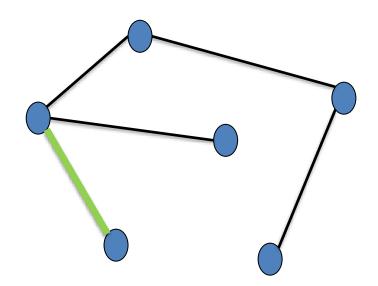
Design and Analysis of Algorithms

Problem: Given a **tree** graph, compute/find a maximum matching. Do not use Maxflow, but directly Greedy.

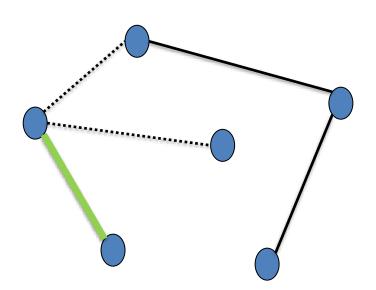


Question: The green edge has a **leaf** as an endpoint. Should it be in the matching?

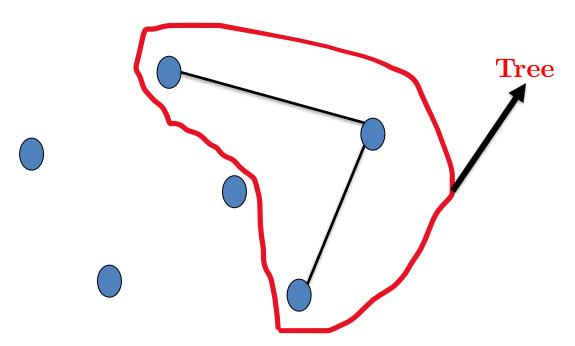
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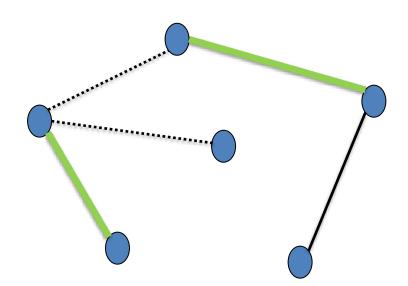
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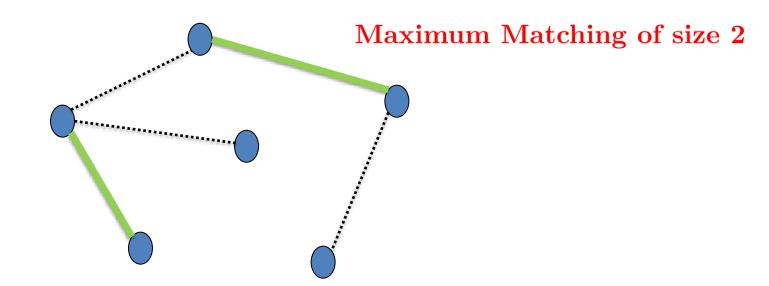
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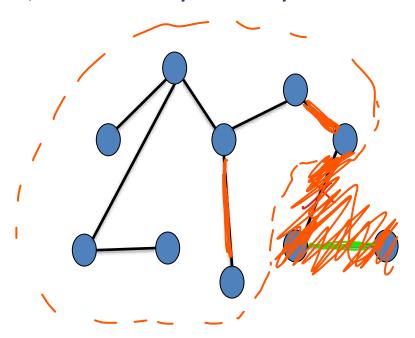


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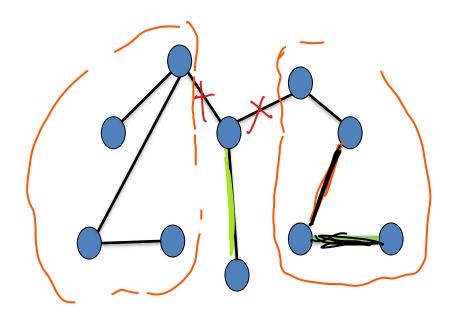


OF trees

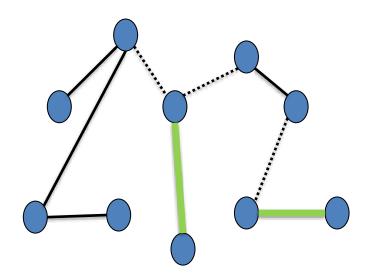
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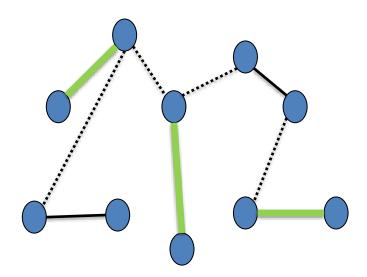
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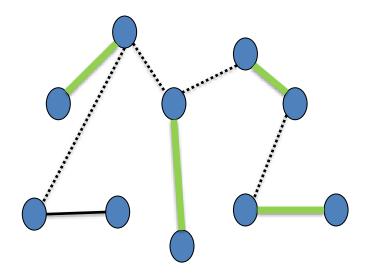
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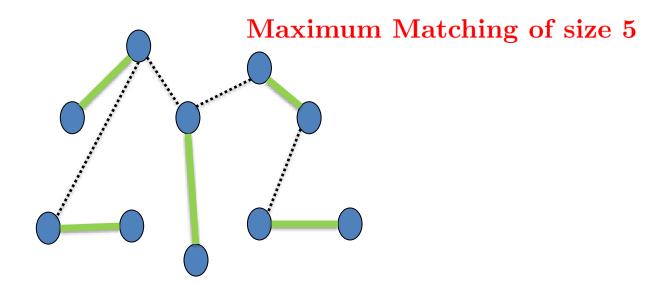
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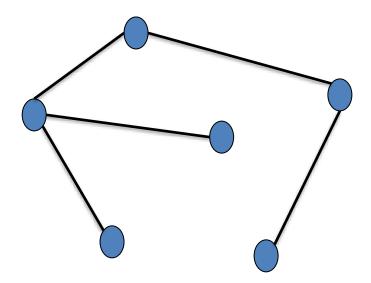
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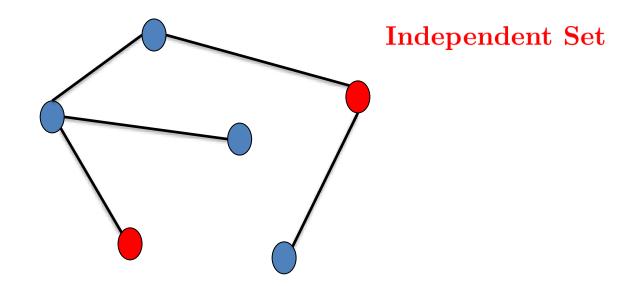
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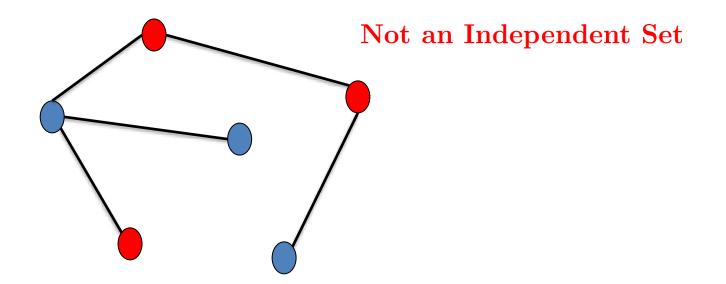
Definition: Given a graph G, an **independent set** is a collection of vertices that do not share an edge.



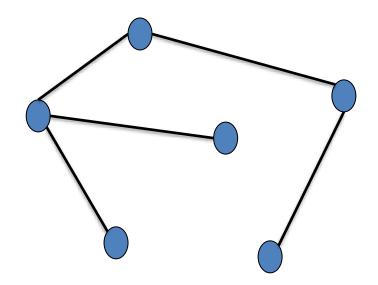
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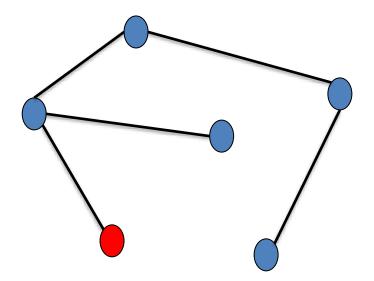


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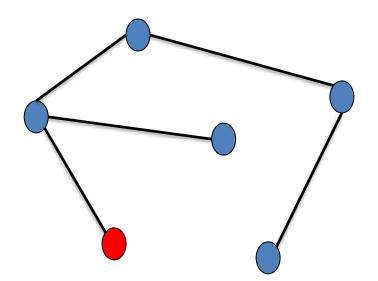
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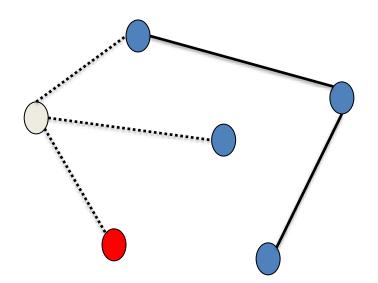


Question: Should a **leaf** be part of the independent set?

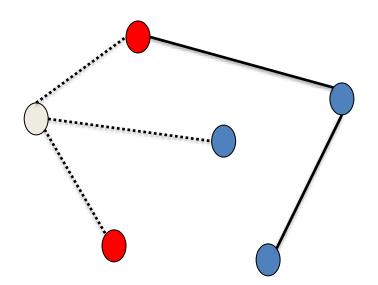
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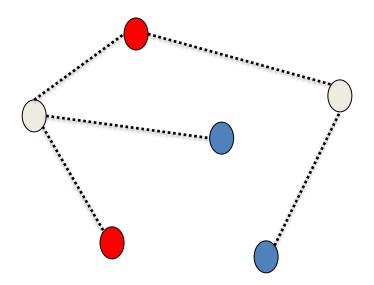
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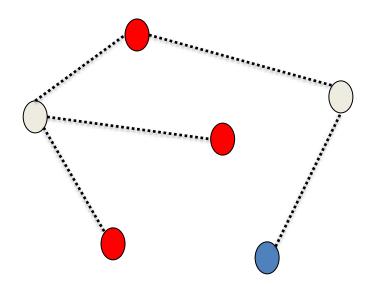
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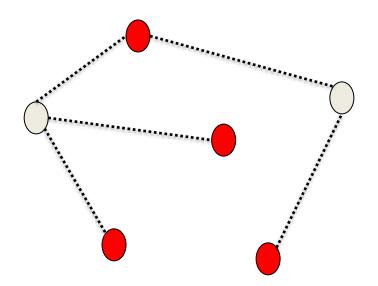
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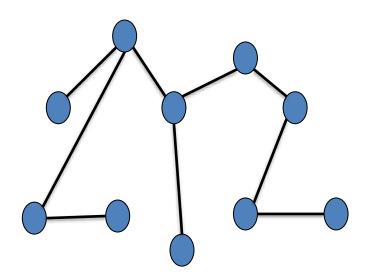
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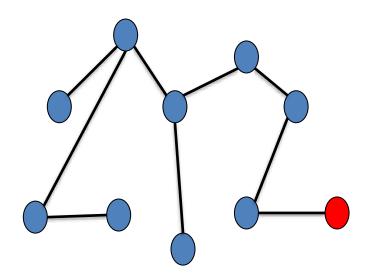
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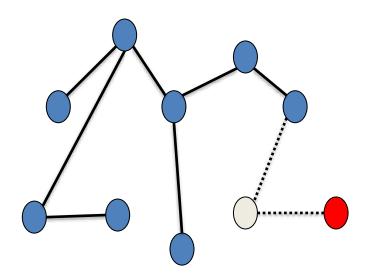
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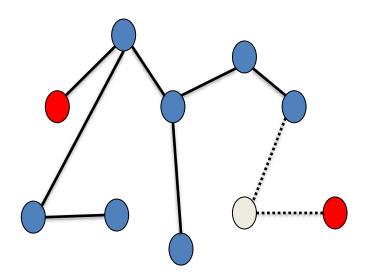
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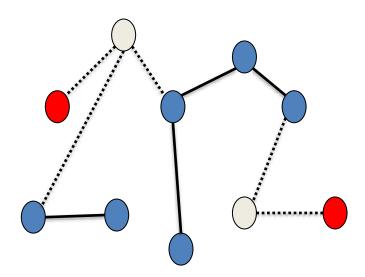
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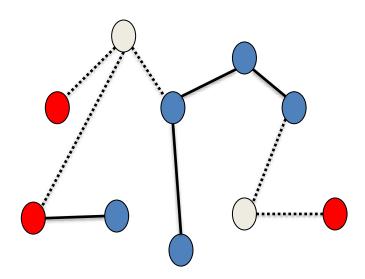
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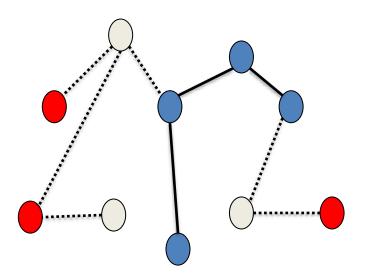
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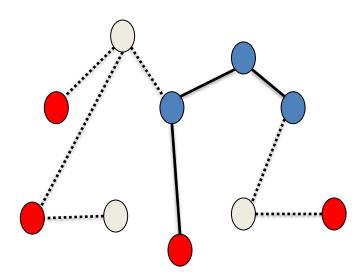
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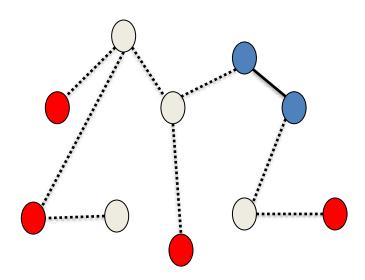
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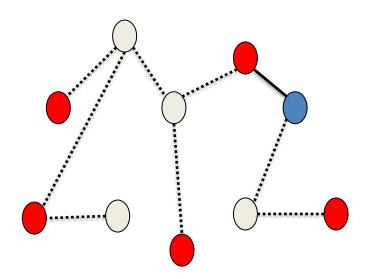
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