Lecture 16

More problems on the Greedy Method

CS 161 Design and Analysis of Algorithms

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Greedy method

The greedy method is a general algorithm design technique, in which given:

- configurations: different choices we need to make
- objective function: a score assigned to all configurations, which we want to either maximize or minimize

We should make choices greedily: We can find a globally-optimal solution by a series of local improvements from a starting configuration.

Scheduling jobs/tasks

Problem 1: Given: a set T of n tasks, each having a start time s_i and a finish time f_i (where $s_i < f_i$)
Goal: Perform all the tasks using a minimum number of machines.
A machine can serve one task at a given time.

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A machine can serve one task at a given time.

Idea: Sort tasks in increasing order of their **start** time. Assign first task to machine 1 and set $K = 1$.
When considering a new task, if all machines are busy, create a new machine, set $K = K + 1$ and assign the new task to the new machine otherwise assign the new task to an available machine.

Scheduling jobs/tasks

Problem 2: Given: a set T of n tasks, each having a start time s_i and a finish time f_i (where $s_i < f_i$)
Goal: Perform as many tasks as possible using one machine.
In other words, find the maximum number of non-overlapping intervals.

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Idea: Sort tasks in increasing order of their **finish** time. Perform first task and remove all overlapping tasks with first task. Repeat the same process to the remaining tasks.

Example: 7 Tasks, [1,4], [1,3], [2,5], [3,7], [4,7], [6,9], [7,8]

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Scheduling jobs/tasks

Problem 3: You are given a set T of n tasks, each having a deadline time f_i and profit p_i if completed and needs one unit of time to be completed. You have only one machine.

Goal: Complete non-overlapping tasks to maximize your profit.

Example: 7 Tasks, (5\$, 3), (2\$, 2), (2\$, 2), (1\$, 2), (4\$, 4), (2\$, 4), (1\$, 4)

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Solution : Task 2, Task 3, Task 1, Task 5 with profit 13\$

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Idea: Sort tasks in decreasing order of their **profit**. Repeat the following until run out of tasks: Choose first task and schedule it at the latest time possible without exceeding deadline. If not possible, discard the task.

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(4\$, 4), (2\$, 2), (2\$, 2), (2\$, 4), (1\$, 2), (1\$, 4)

Schedule task (5\$,3) between times 2-3.

Scheduling jobs/tasks

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Solution : Task 2, Task 3, Task 1, Task 5 with profit 13\$

Idea: Sort tasks in decreasing order of their **profit**. Repeat the following until run out of tasks: Choose first task and schedule it at the latest time possible without exceeding deadline. If not possible, discard the task.

(2\$, 2), (2\$, 2), (2\$, 4), (1\$, 2), (1\$, 4)

Schedule task (5\$,3) between times 2-3. Schedule task (4\$,4) between times 3-4.

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Solution : Task 2, Task 3, Task 1, Task 5 with profit 13\$

Idea: Sort tasks in decreasing order of their **profit**. Repeat the following until run out of tasks: Choose first task and schedule it at the latest time possible without exceeding deadline. If not possible, discard the task.

(2\$, 2), (2\$, 4), (1\$, 2), (1\$, 4)

Schedule task (5\$,3) between times 2-3. Schedule task (4\$,4) between times 3-4. Schedule task (2\$,2) between times 1-2.

Scheduling jobs/tasks

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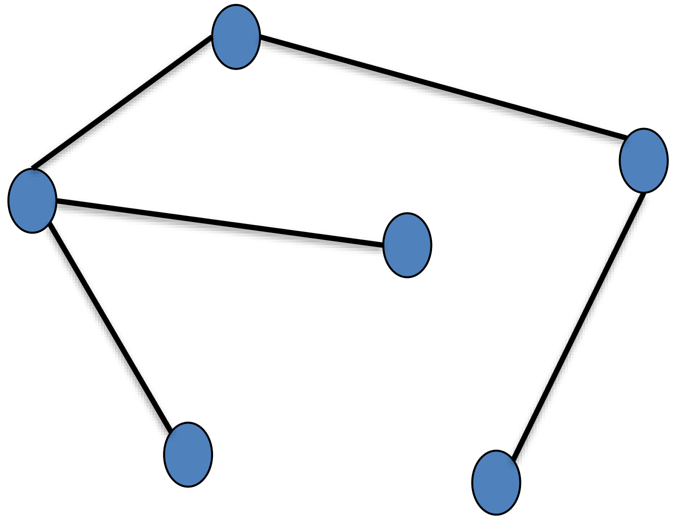
(2\$, 4), (1\$, 2), (1\$, 4)

Schedule task (5\$,3) between times 2-3. Schedule task (4\$,4) between times 3-4. Schedule task (2\$,2) between times 1-2. Schedule task (2\$,2) between times 0-1.

Problems on trees using Greedy

Definition: Given a graph G, a **matching** is a collection of edges that do not share a vertex.



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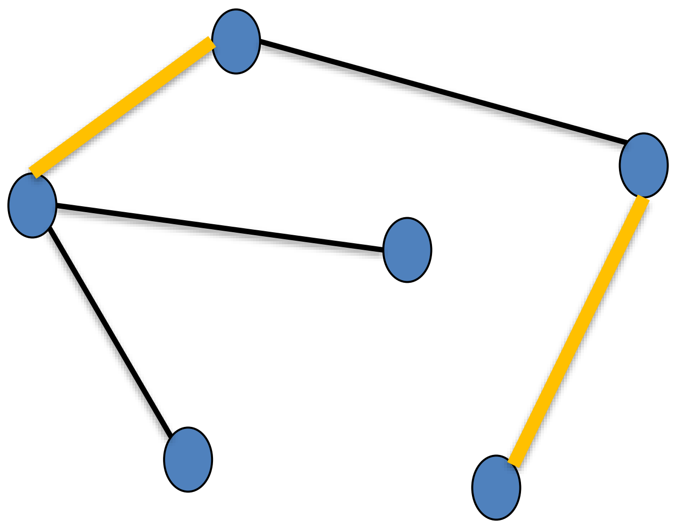

Matching

Problems on trees using Greedy

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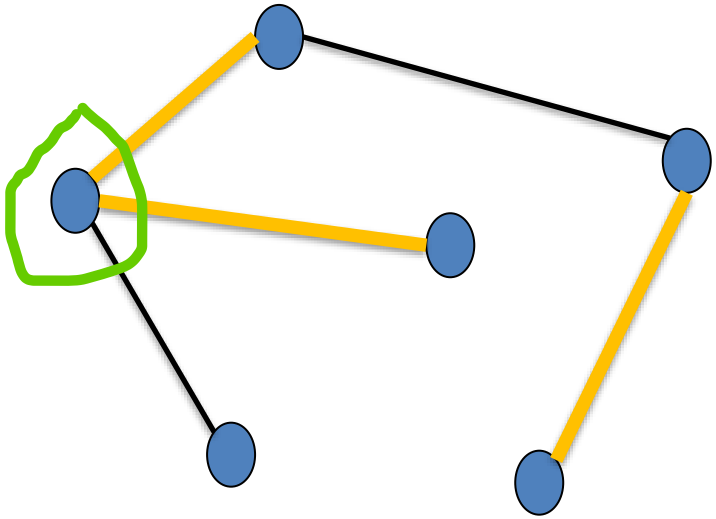

Not a Matching

Problems on trees using Greedy

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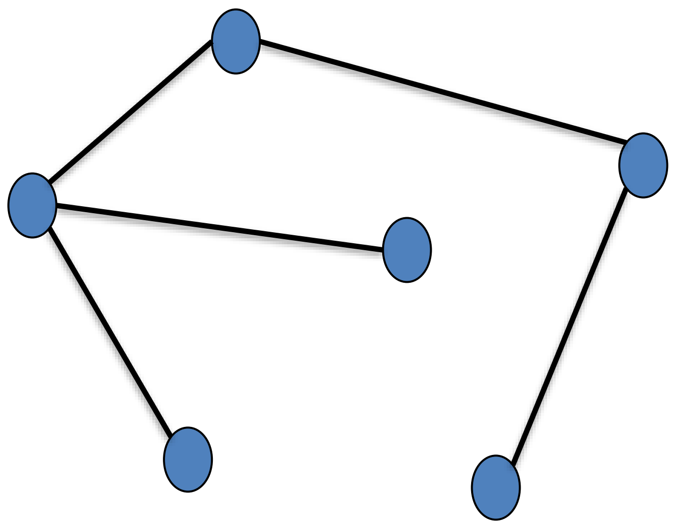

Problem: Given a **tree** graph, compute/find a maximum matching.

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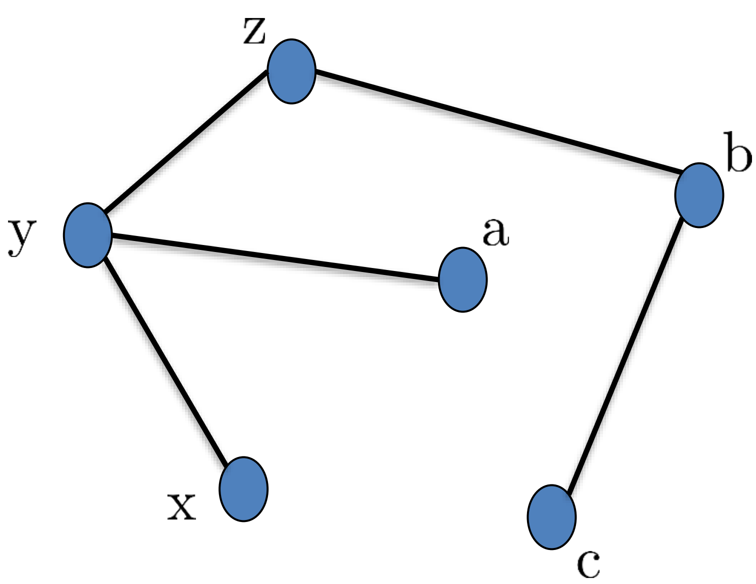

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We know how to do it for bipartite graphs via maxflow!!

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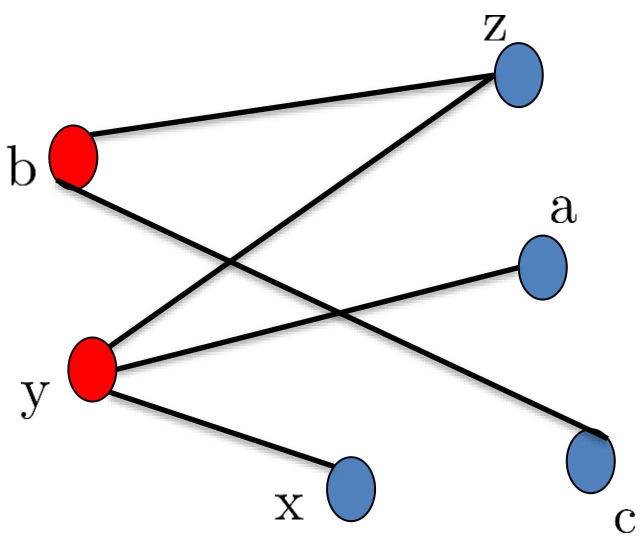
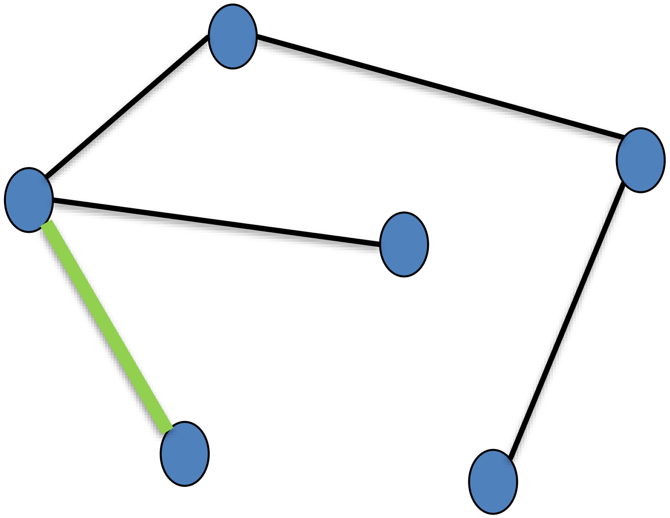


Problem: Given a **tree** graph, compute/find a maximum matching.
We know how to do it for bipartite graphs via maxflow!!

Trees are bipartite!

Maximum Matching on trees

Problem: Given a **tree** graph, compute/find a maximum matching. Do not use Maxflow, but directly Greedy.



Question: The green edge has a **leaf** as an endpoint. Should it be in the matching?

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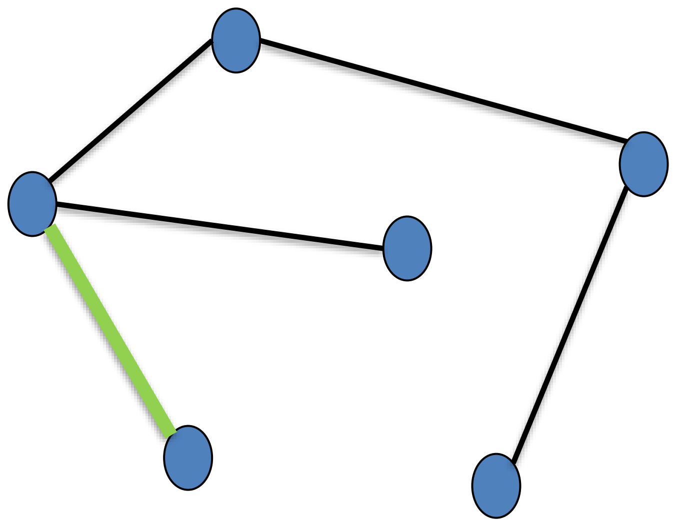

Idea: Choose an edge with one endpoint being a leaf and put it in the matching. Remove all other incident edges. The new graph is a union of trees. Repeat until run out of edges.

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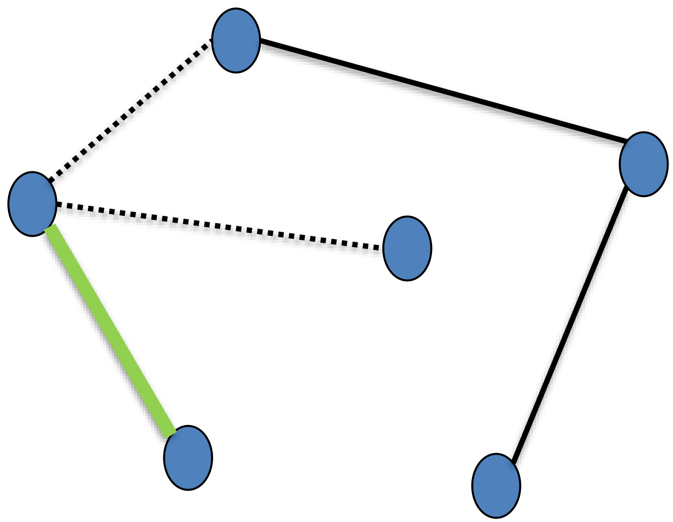

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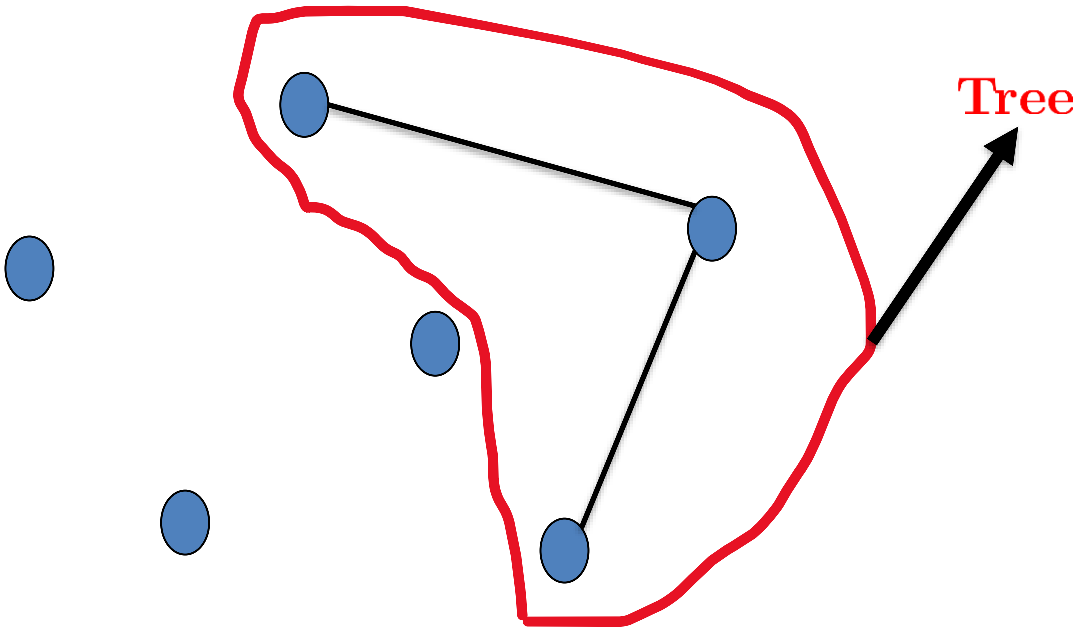

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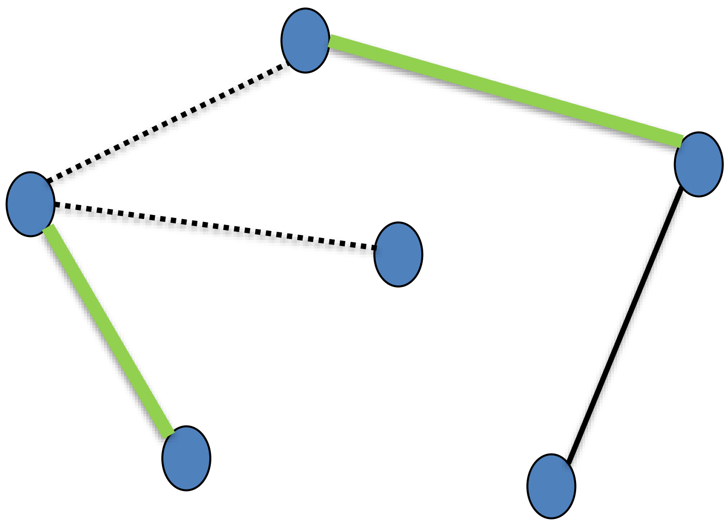

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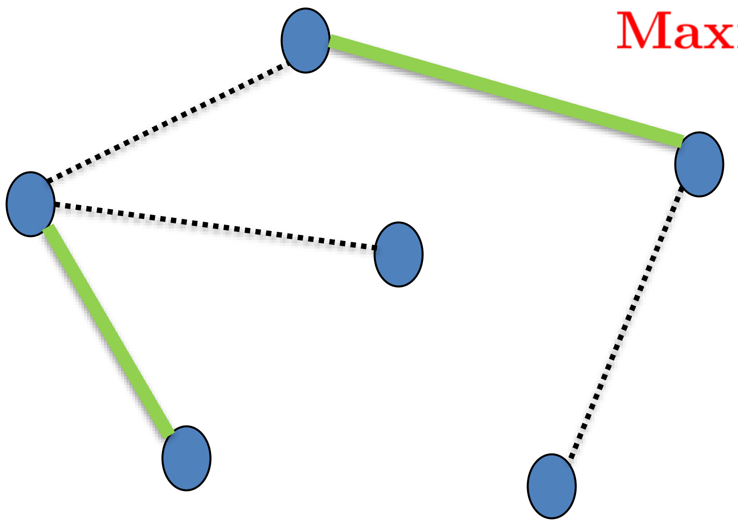

Maximum Matching of size 2

Idea: Choose an edge with one endpoint being a leaf and put it in the matching. Remove all other incident edges. The new graph is a union of trees. Repeat until run out of edges.

Maximum Matching on trees

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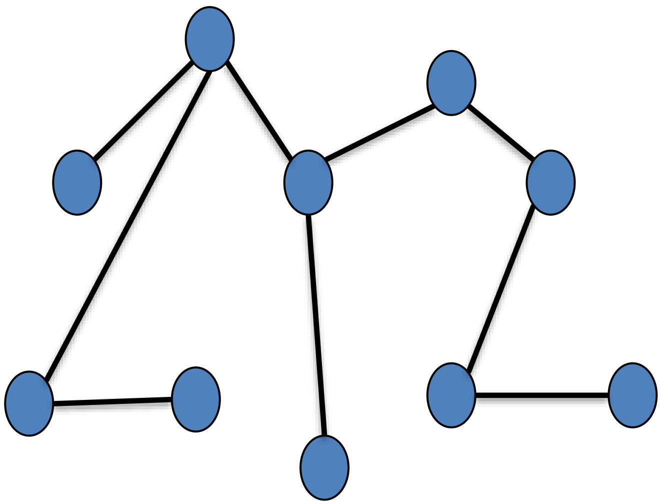

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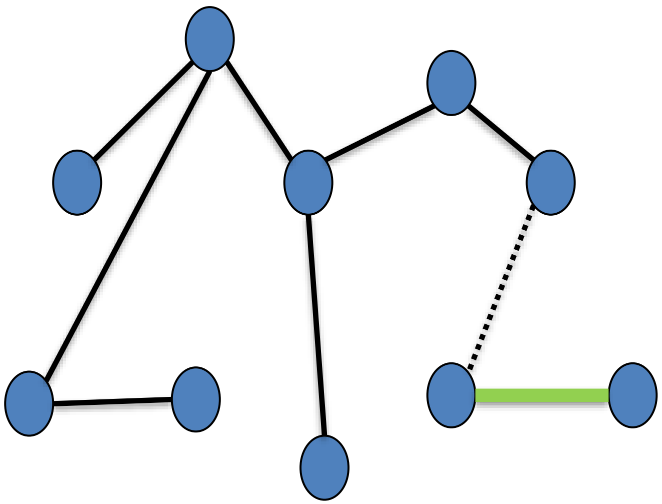

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Maximum Matching on trees

Problem: Given a **tree** graph, compute/find a maximum matching.
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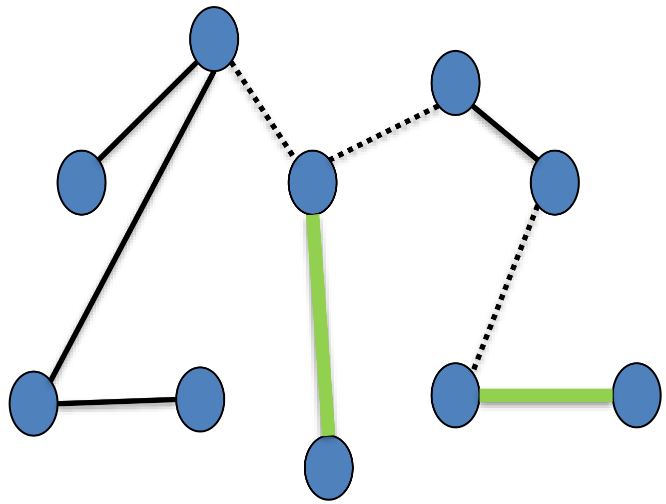

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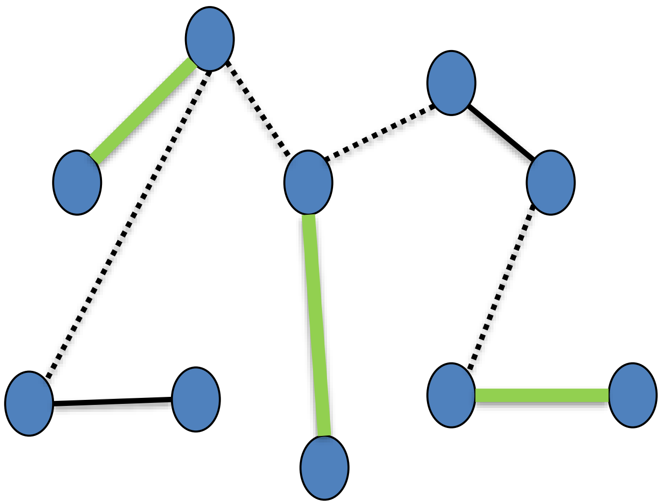

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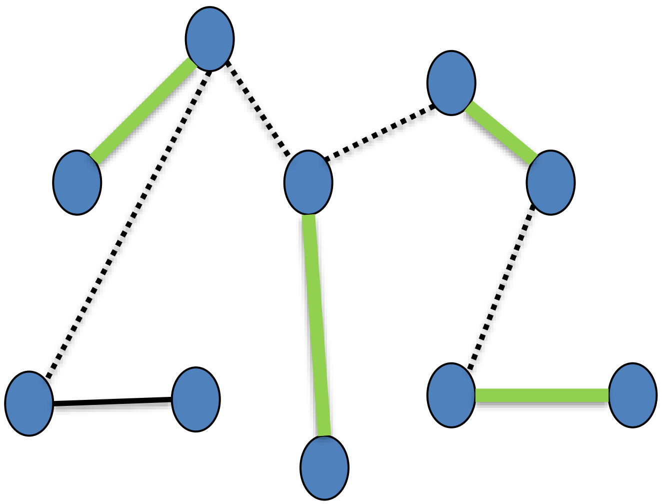

Idea: Choose an edge with one endpoint being a leaf and put it in the matching. Remove all other incident edges. The new graph is a union of trees. Repeat until run out of edges.

Maximum Matching on trees

Problem: Given a **tree** graph, compute/find a maximum matching.
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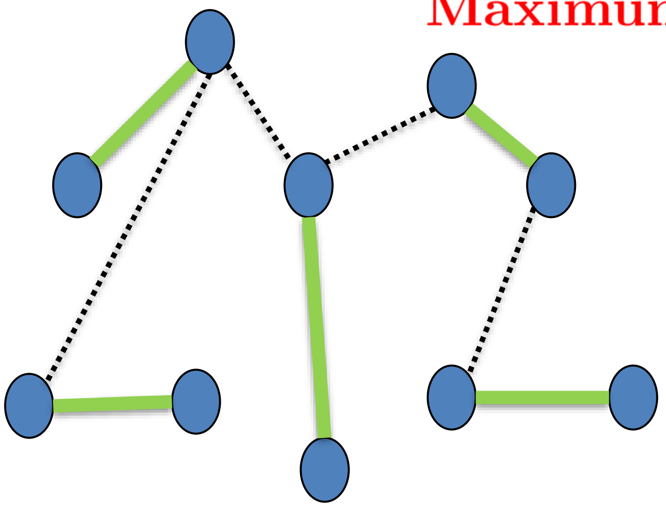
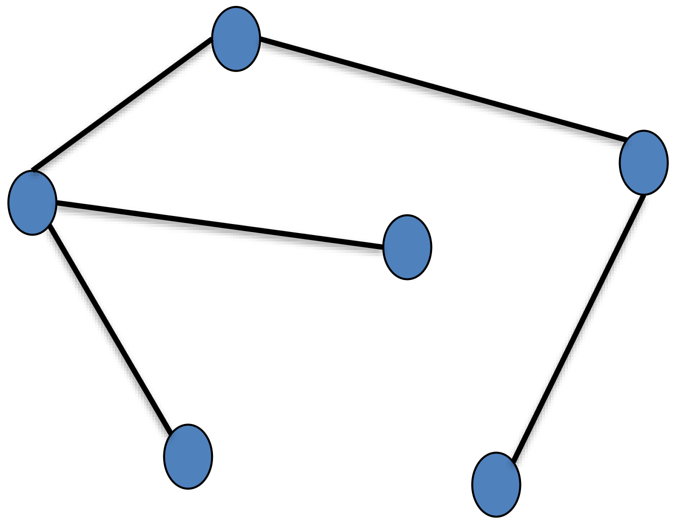
Maximum Matching of size 5



Idea: Choose an edge with one endpoint being a leaf and put it in the matching. Remove all other incident edges. The new graph is a union of trees. Repeat until run out of edges.

Problems on trees using Greedy

Definition: Given a graph G, an **independent set** is a collection of vertices that do not share an edge.



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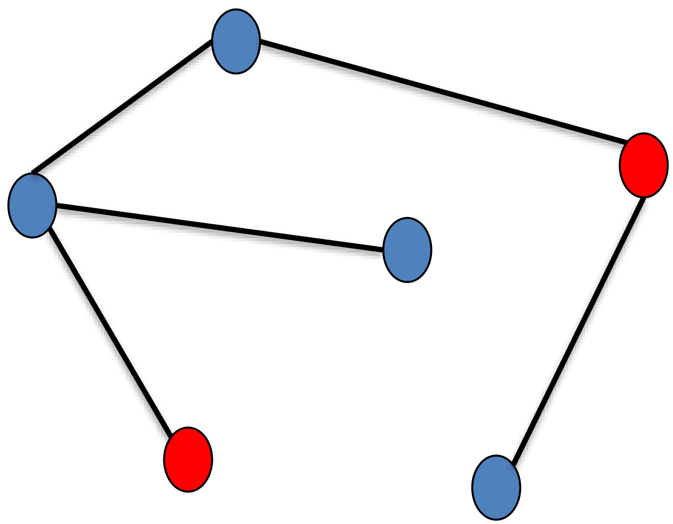

Independent Set

Problems on trees using Greedy

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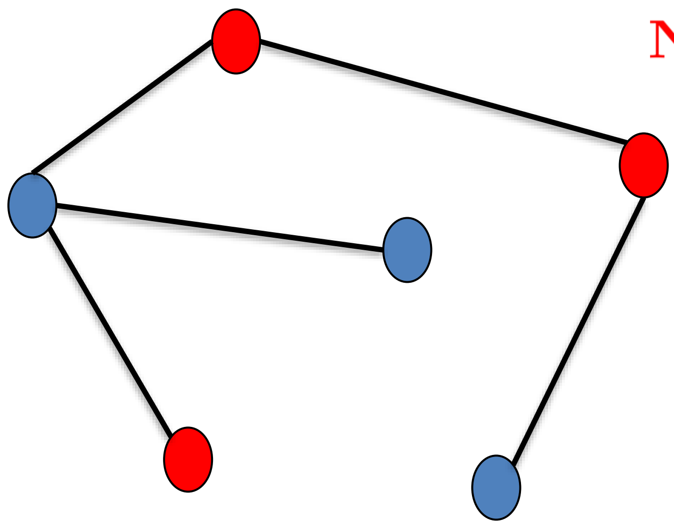

Not an Independent Set

Problems on trees using Greedy

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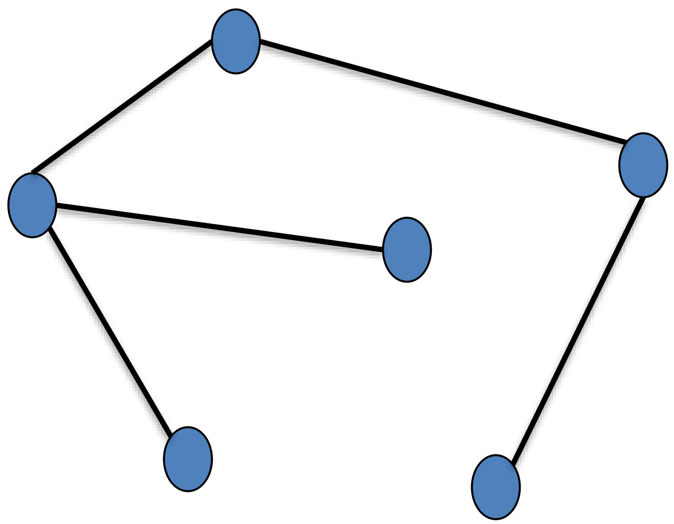

Problem: Given a **tree** graph, compute/find a maximum independent set.

Maximum Independent set on trees

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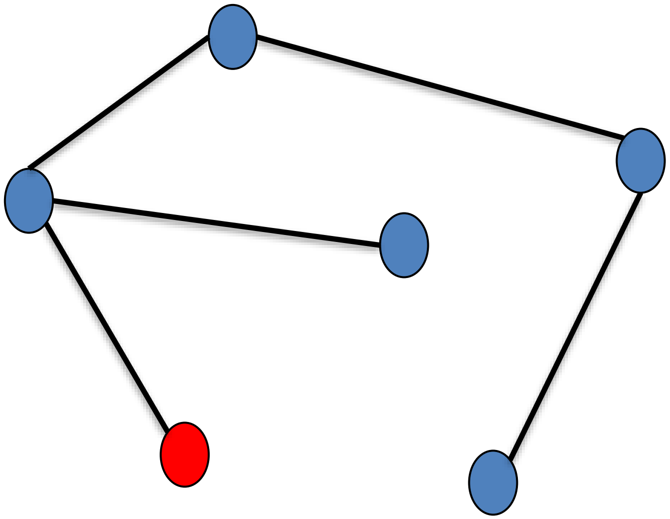
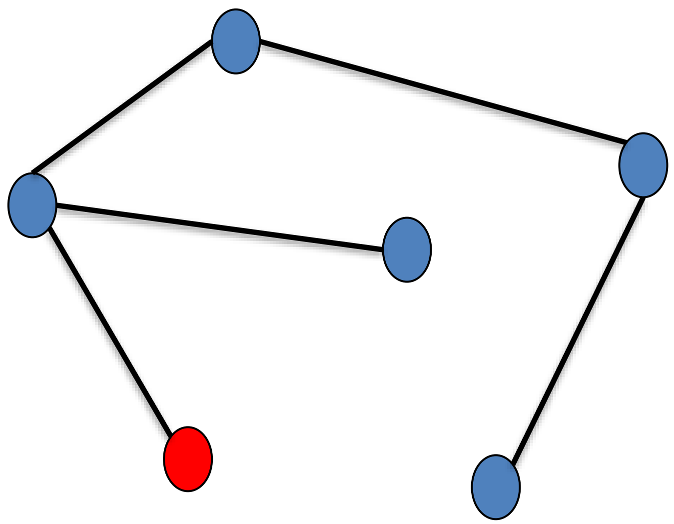


Question: Should a **leaf** be part of the independent set?

Maximum Independent set on trees

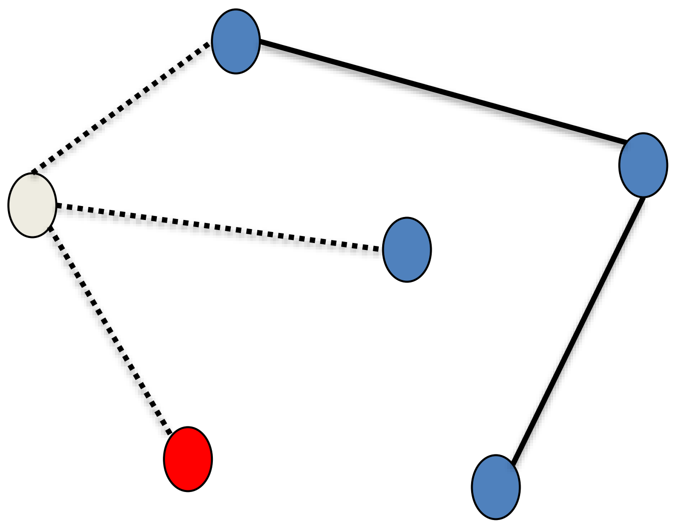
Problem: Given a **tree** graph, compute/find a maximum independent set.



Idea: Choose a leaf (or isolated vertex) and put it in the independent set. Remove the neighboring vertex (and incident edges). The new graph is a union of trees. Repeat until run out of vertices.

Maximum Independent set on trees

Problem: Given a **tree** graph, compute/find a maximum independent set.



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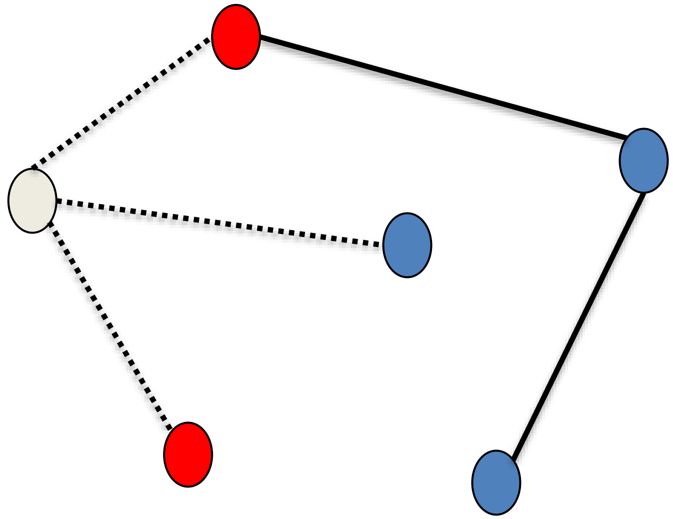

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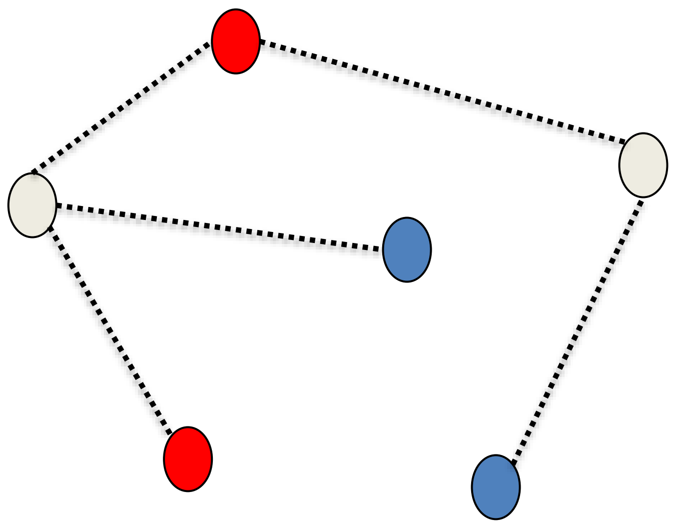

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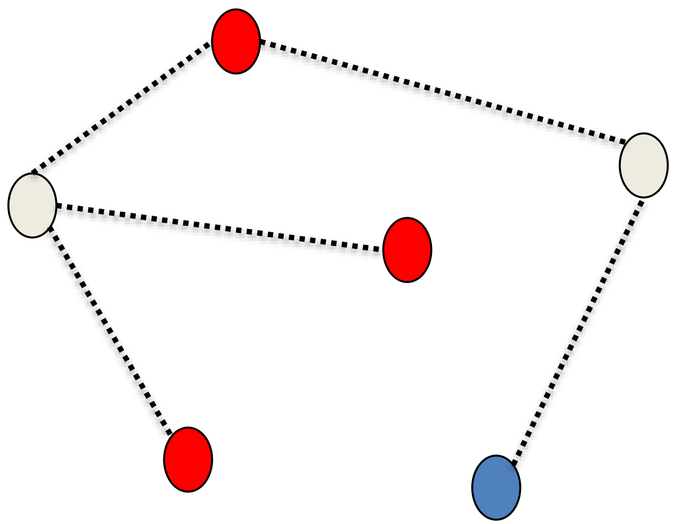

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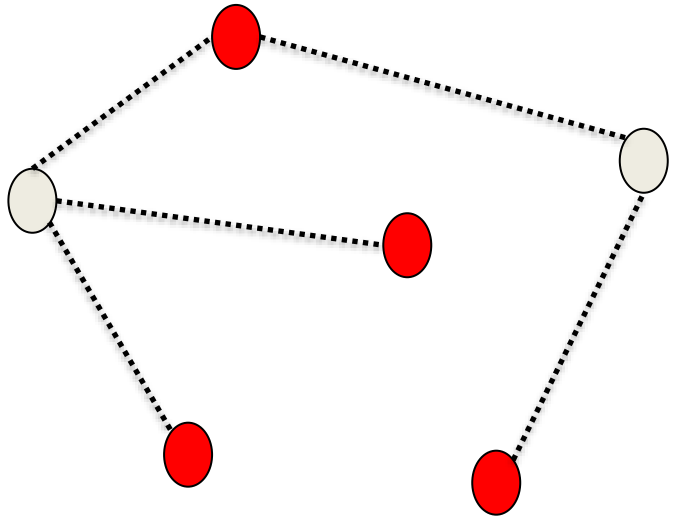

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Maximum Independent set on trees

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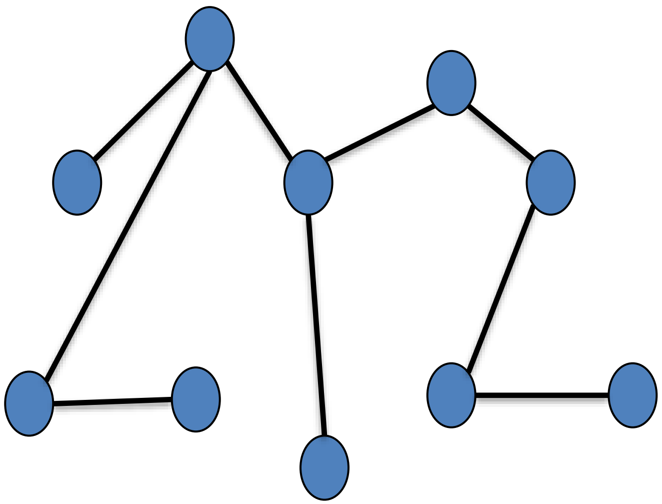

Idea: Choose a leaf (or isolated vertex) and put it in the independent set. Remove the neighboring vertex (and incident edges). The new graph is a union of trees. Repeat until run out of vertices.

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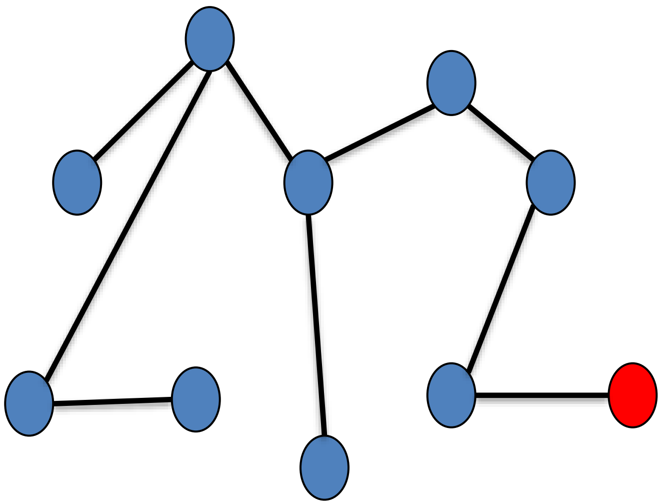

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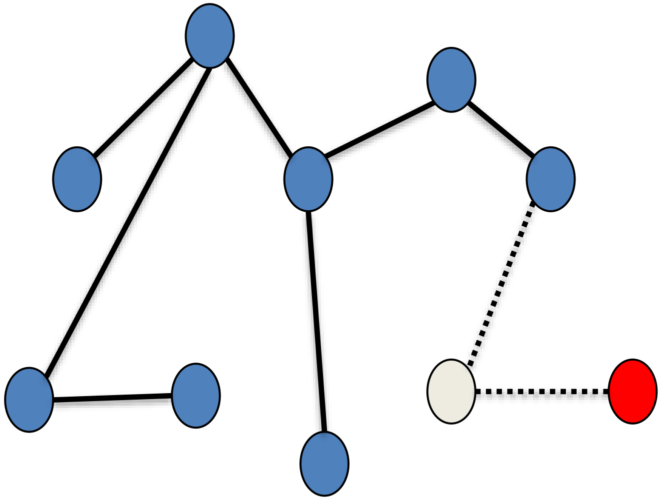

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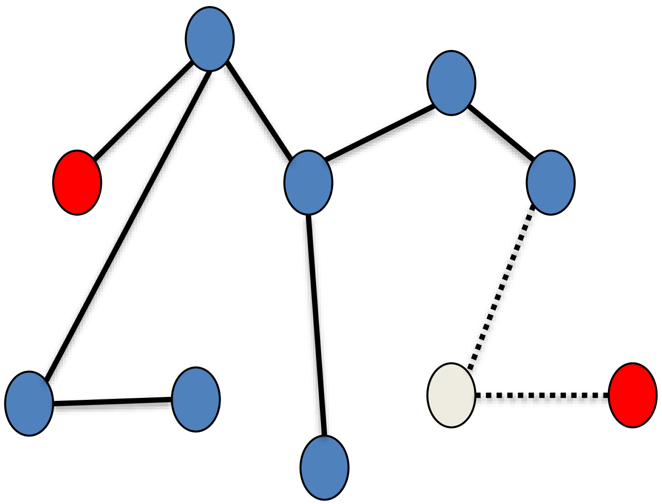

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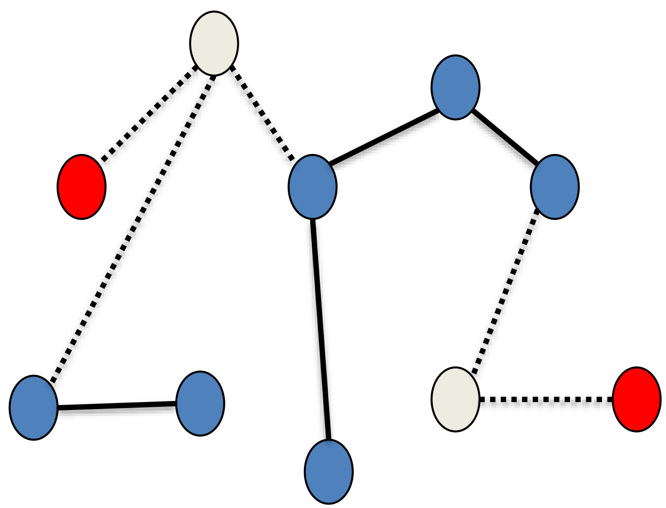

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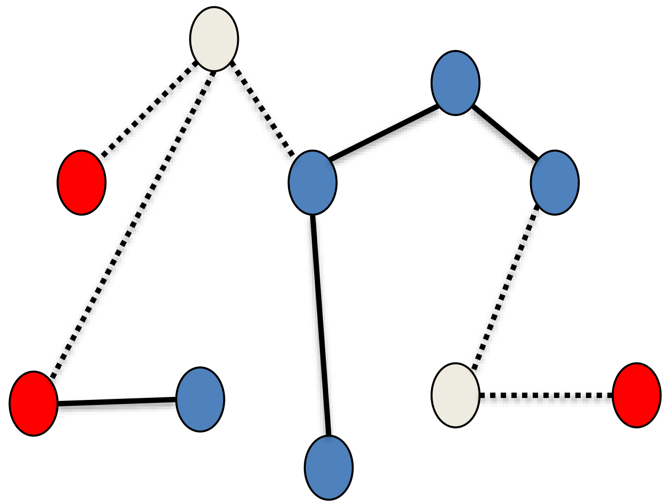

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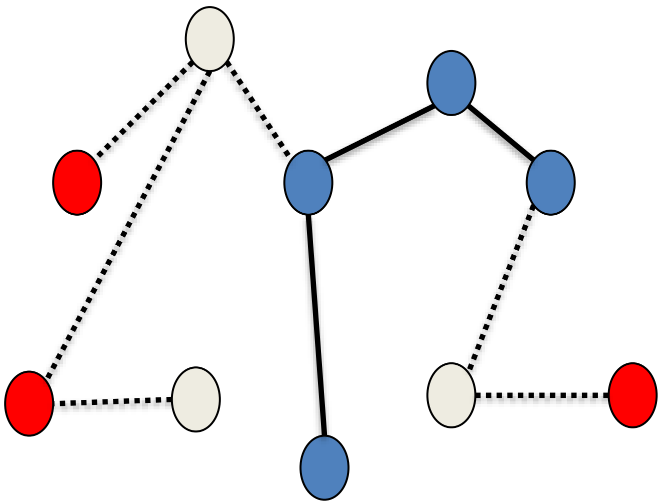

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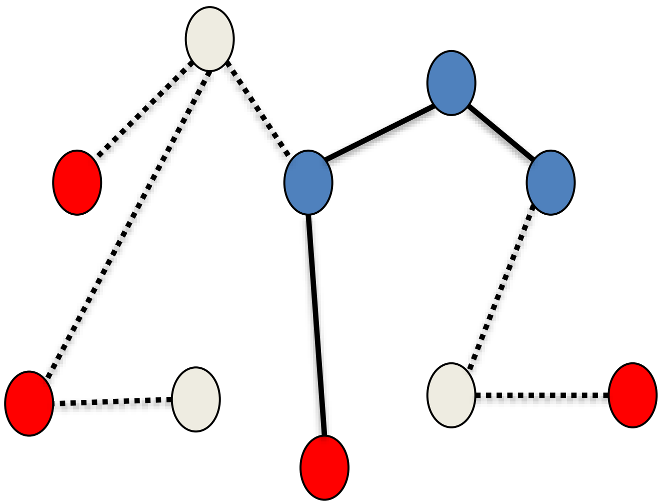

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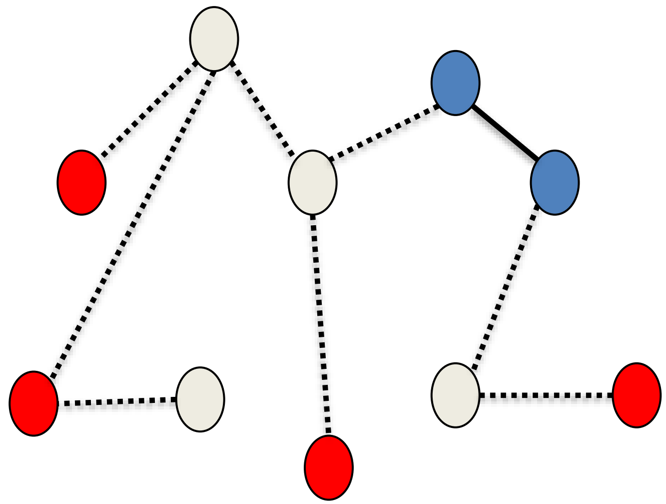

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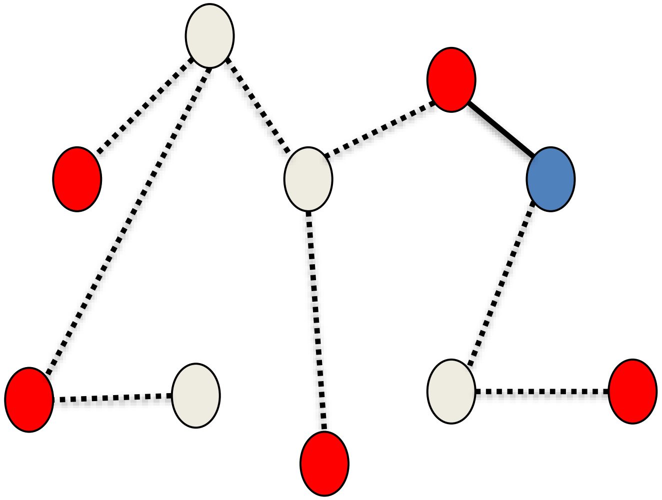

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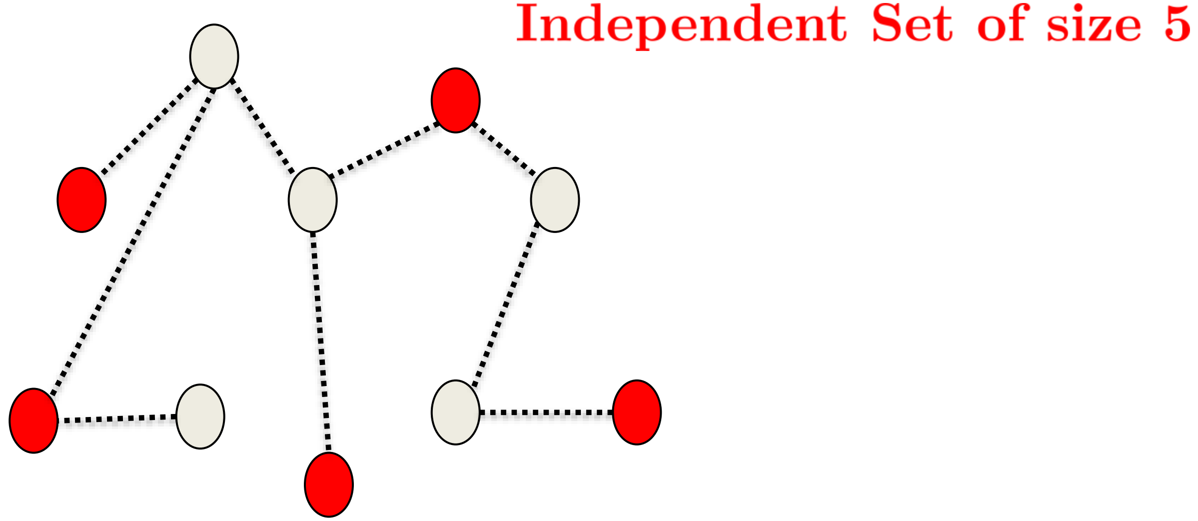

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