Lecture 4

Divide and Conquer II: Counting inversions, counting intersections, max subarray, maxima set

CS 161 Design and Analysis of Algorithms

Ioannis Panageas

Divide and conquer method (recap)

Steps of method:

- **Divide** input into parts (smaller problems)
- **Conquer** (solve) each part recursively
- **Combine** results to obtain solution of original

$$T(n) = \text{divide time} + T(n_1) + T(n_2) + \ldots + T(n_k) + \text{combine time}$$

Case study I: Counting inversions

Given numbers $A_1, \ldots, A_n$ in an array A, compute the number of inversions.

(i, j) is an inversion: $A_i > A_j$ and $i < j$.

Example [18, 29, 12, 15, 32, 10] has 9 inversions:

(18,12), (18,15), (18,10), (29,12), (29,15), (29,10), (12,10), (15,10), (32,10)

Case study I: Counting inversions

- Solution: Use Divide and conquer. Tricky part the combine step.

Run a modification of Mergesort that has a counter that counts inversions during merge steps.

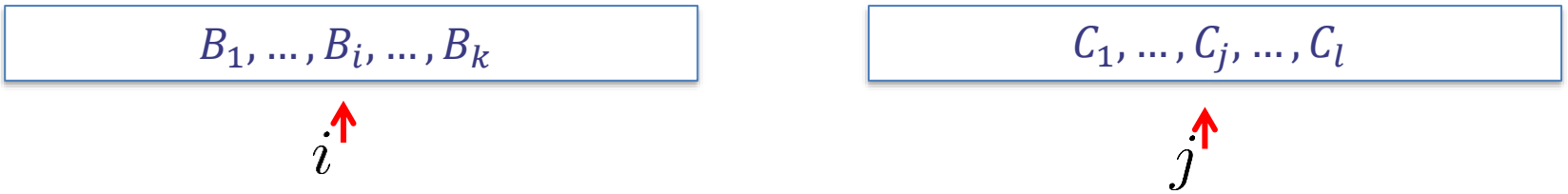
- Question: Assume that $B_1, \dots, B_k$ and $C_1, \dots, C_l$ are both sorted. Can you compute the number of inversions of the concatenated sequence $B_1, \dots, B_k, C_1, \dots, C_l$?

Case study I: Counting inversions

- Question: Assume that $B_1, \dots, B_k$ and $C_1, \dots, C_l$ are both sorted. Can you compute the number of inversions of the sequence $B_1, \dots, B_k, C_1, \dots, C_l$?

If $B_i > C_j \geq B_{i-1}$ there are

including C_j

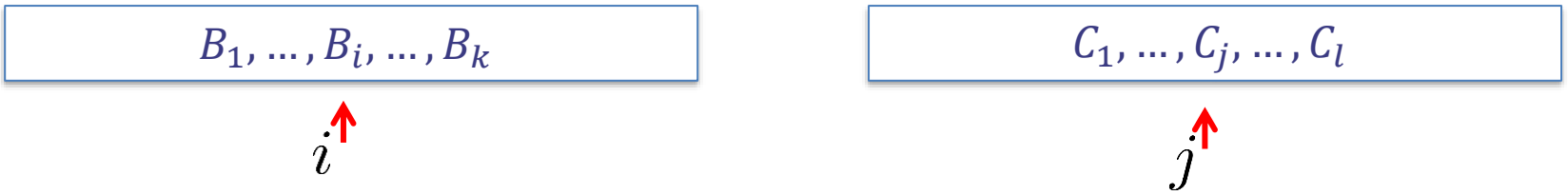


Case study I: Counting inversions

- Question: Assume that $B_1, \dots, B_k$ and $C_1, \dots, C_l$ are both sorted. Can you compute the number of inversions of the sequence $B_1, \dots, B_k, C_1, \dots, C_l$?

If $B_i > C_j \geq B_{i-1}$ there are

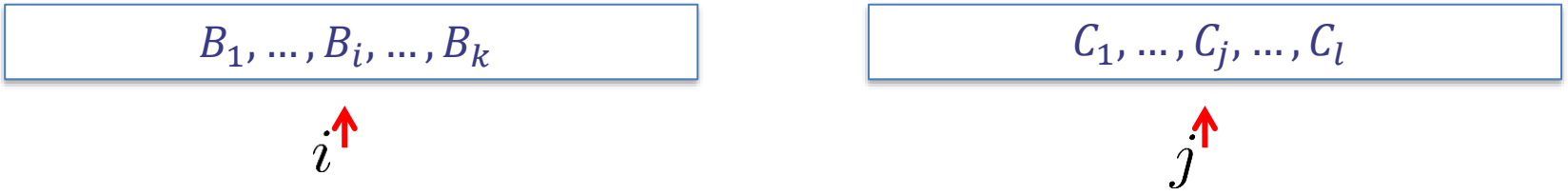
$k - i + 1$ including C_j



Case study I: Counting inversions

If $B_i > C_j \geq B_{i-1}$ there are

$k - i + 1$ including C_j



B : 1 3 4 9 220 C : 2 3 5 7 8 10

Concatenated: 1 3 4 9 220 2 3 5 7 8 10

8 participates in 2 inversions. $k = 5, i = 4$

Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

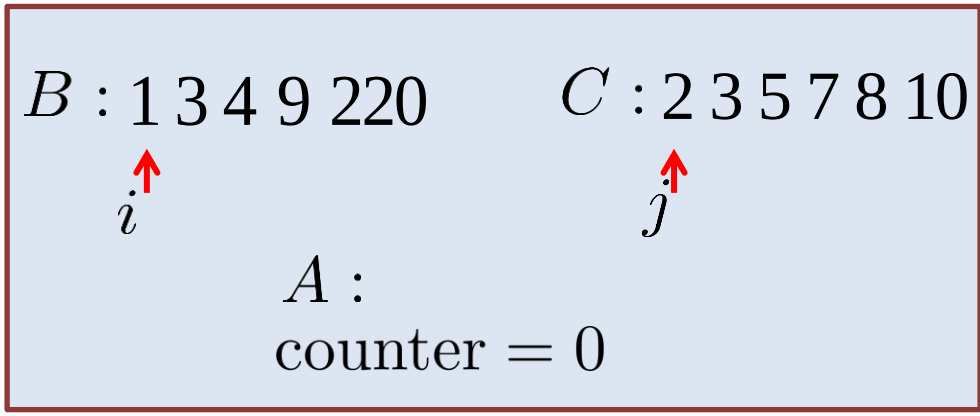
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

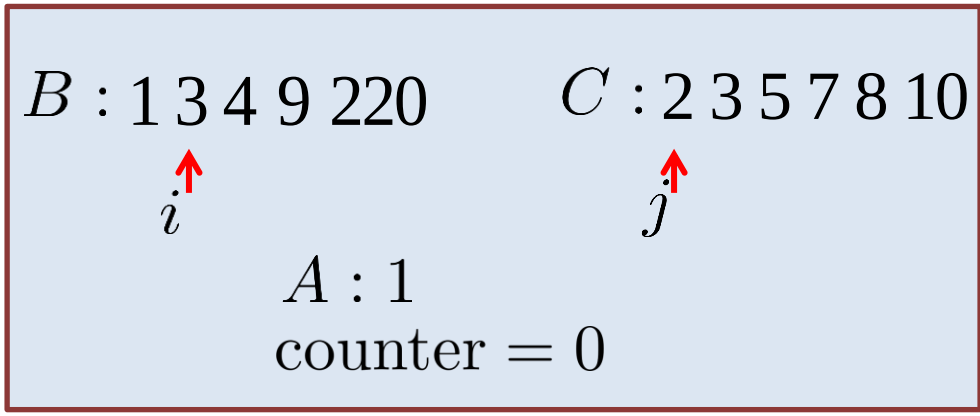
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

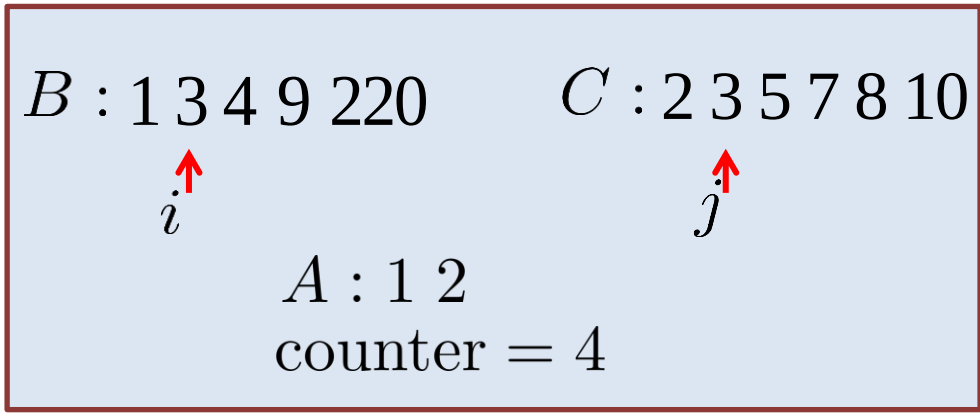
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

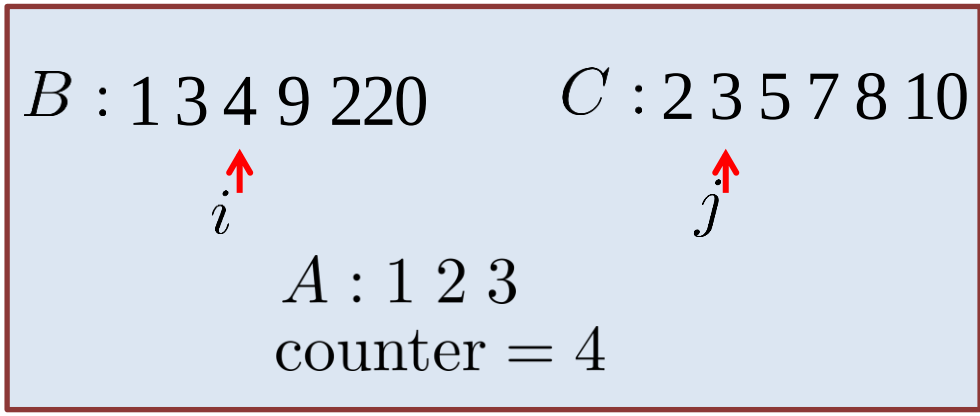
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

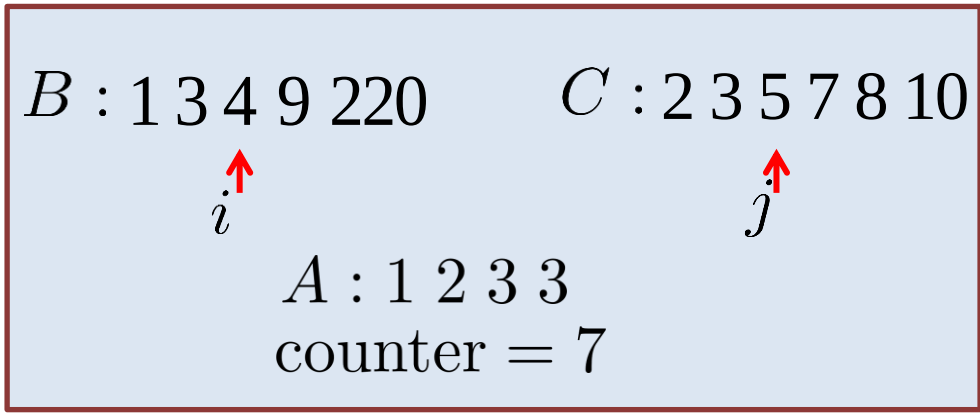
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

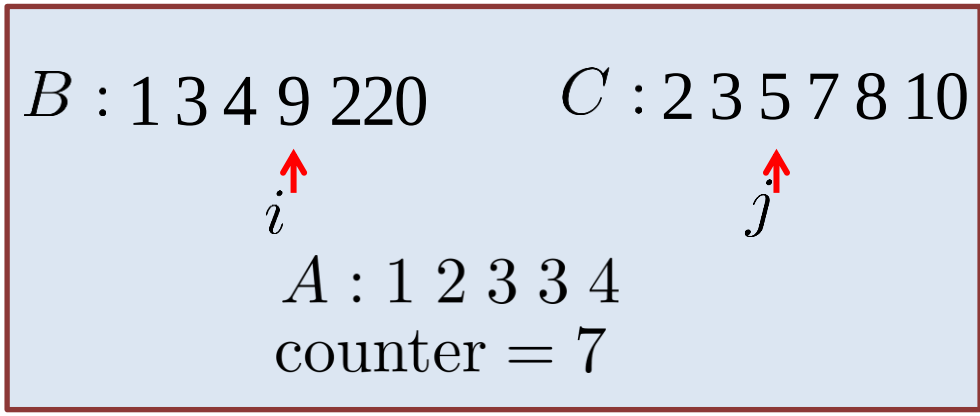
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

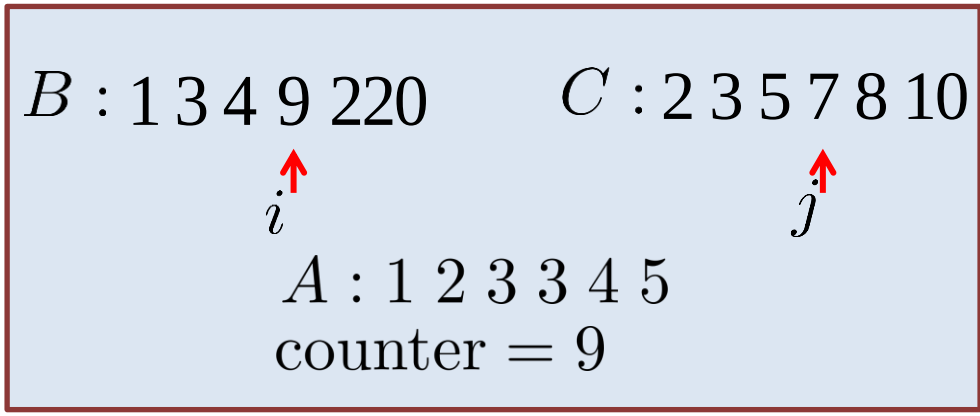
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

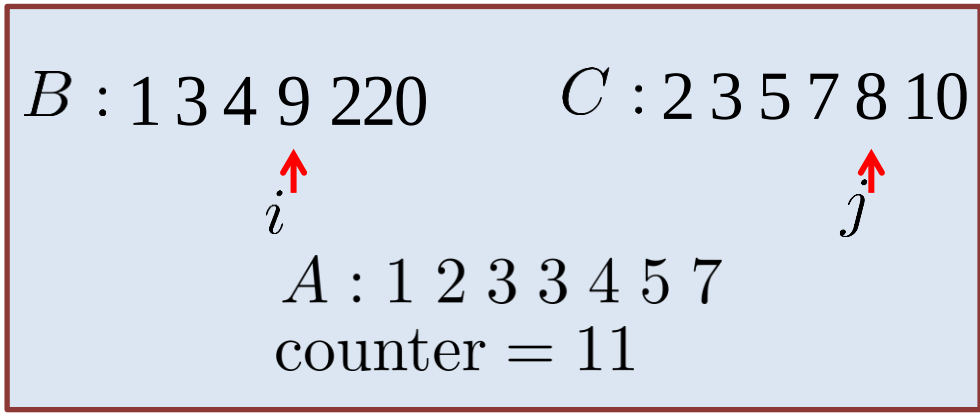
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

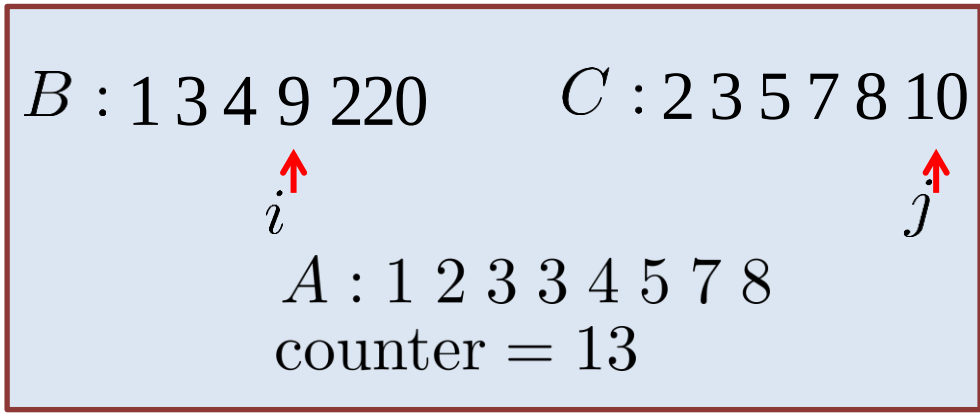
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

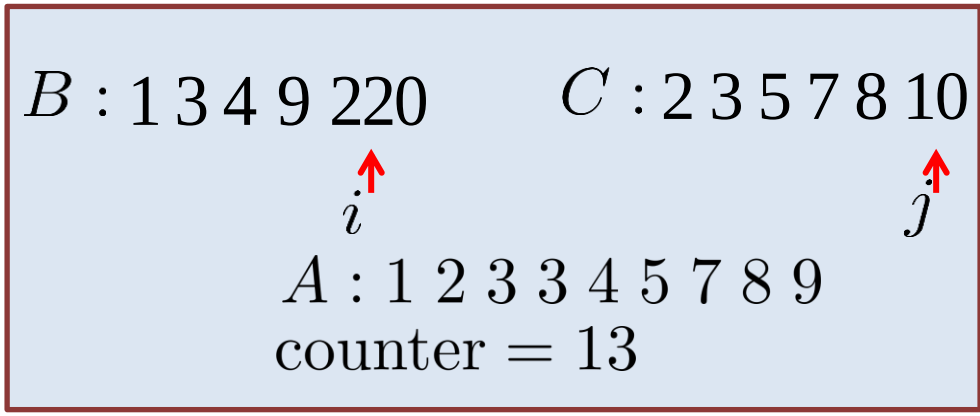
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

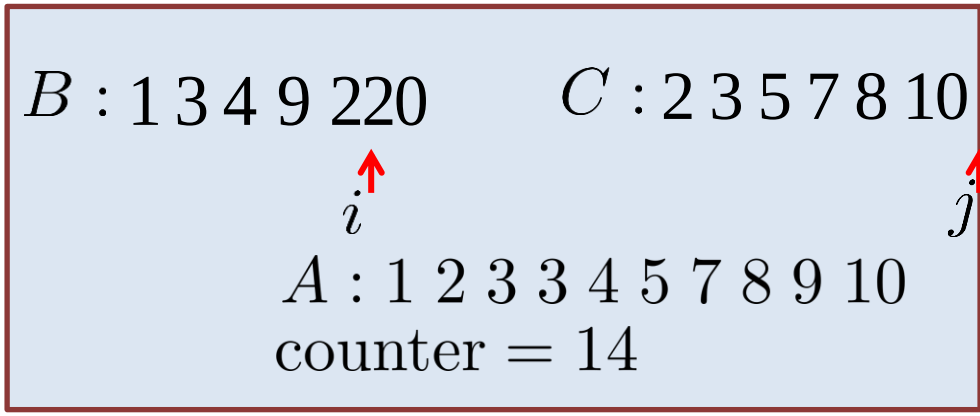
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Problem: Given two sorted arrays B, C, merge them to a sorted array and count number of inversions simultaneously.

Solution: Index i for B, index j for C, index k for A, counter.

While $k < \text{len(B)} + \text{len(C)}$ **do**

  If $B[i] <= C[j]$ **then**

    $A[k] \leftarrow B[i]$

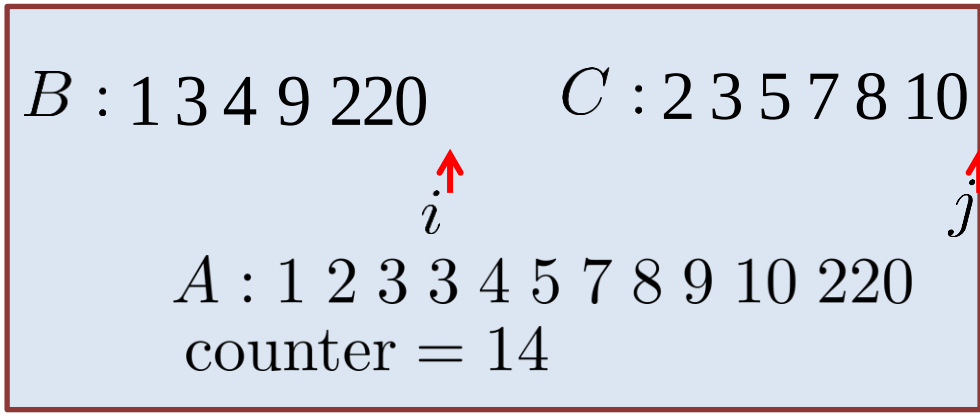
    $i = i + 1, k = k + 1$

  else

    $A[k] \leftarrow C[j]$

    $\text{counter} = \text{counter} + \text{len}(B) - i + 1$

    $j = j + 1, k = k + 1$



Case study I: Counting inversions

Pseudocode:

ModifiedMergesort($A[1:n]$)

If $n == 1$ **then**

return $A, 0$

$B, \mathbf{countL} = \text{ModifiedMergesort}\ (A[1:\frac{n}{2}])$

$C, \mathbf{countR} = \text{ModifiedMergesort}\ (A[\frac{n}{2}+1:n])$

$A, \text{counterM} \leftarrow \text{ModifiedMerge}(B, C)$

return $A, \text{countL} + \text{countR} + \text{counterM}$

$\Theta(1)$

$T(n/2)$

$T(n/2)$

$\Theta(n)$

$T(n) = \Theta(1) + \Theta(n) + 2T(n/2)$

$a = 2, b = 2$

$n^{\log_b a} = n$ $\Rightarrow$ case 2

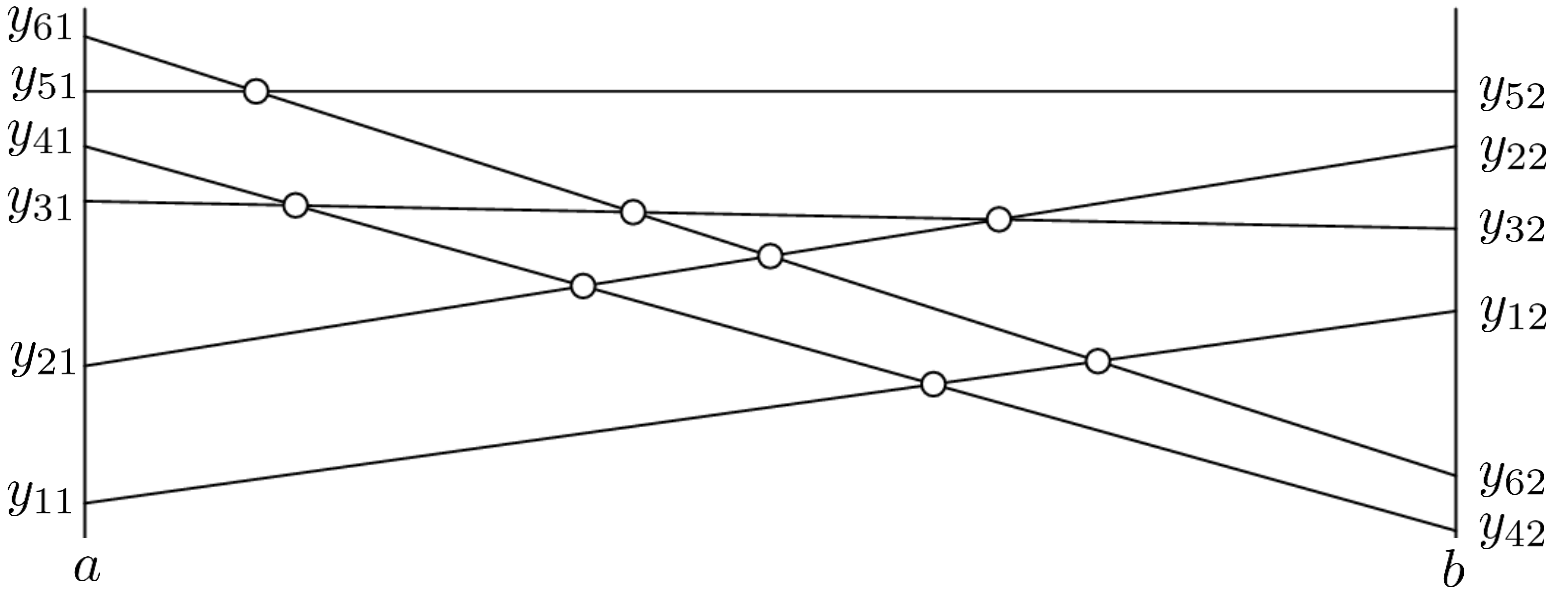
$f(n) = n$

$n \log n$

Case study II: Counting intersections

Problem: Given n distinct lines in the plane, none of which are vertical and two vertical lines $x = a$ and $x = b$, find the number of intersections. We assume that each line i is described by its endpoints (a, y_{i1}) and (b, y_{i2}).

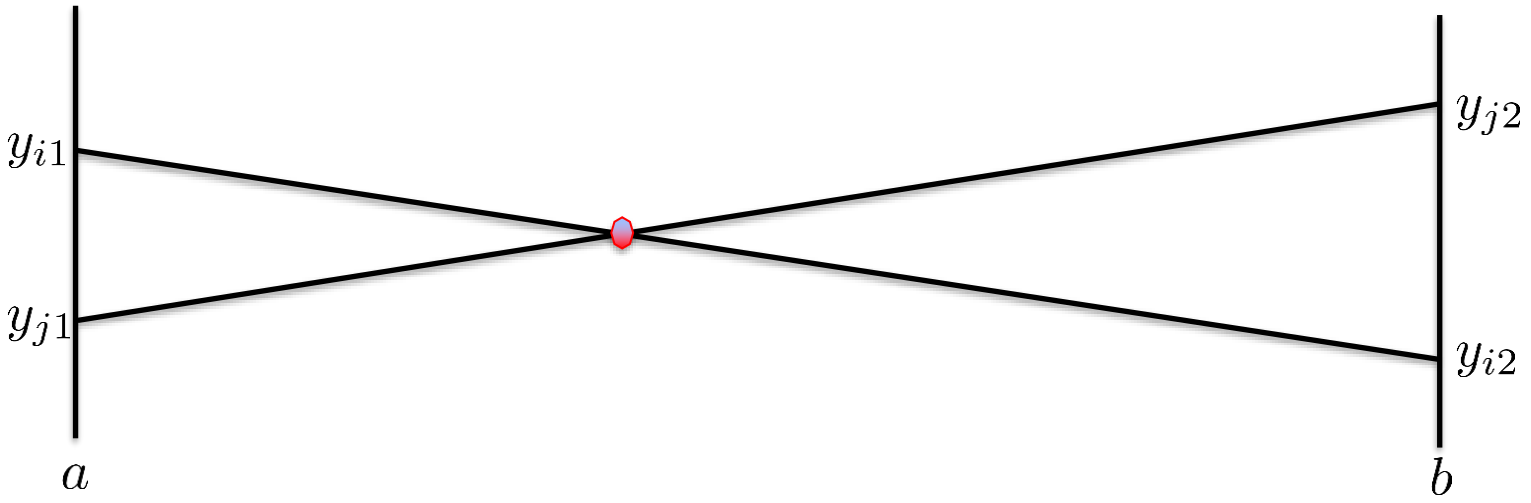
Example: 6 lines (8 intersections)



Case study II: Counting intersections

Problem: Given n distinct lines in the plane, none of which are vertical and two vertical lines $x = a$ and $x = b$, find the number of intersections. We assume that each line i is described by its endpoints (a, y_{i1}) and (b, y_{i2}).

Question: When two lines i, j intersect?

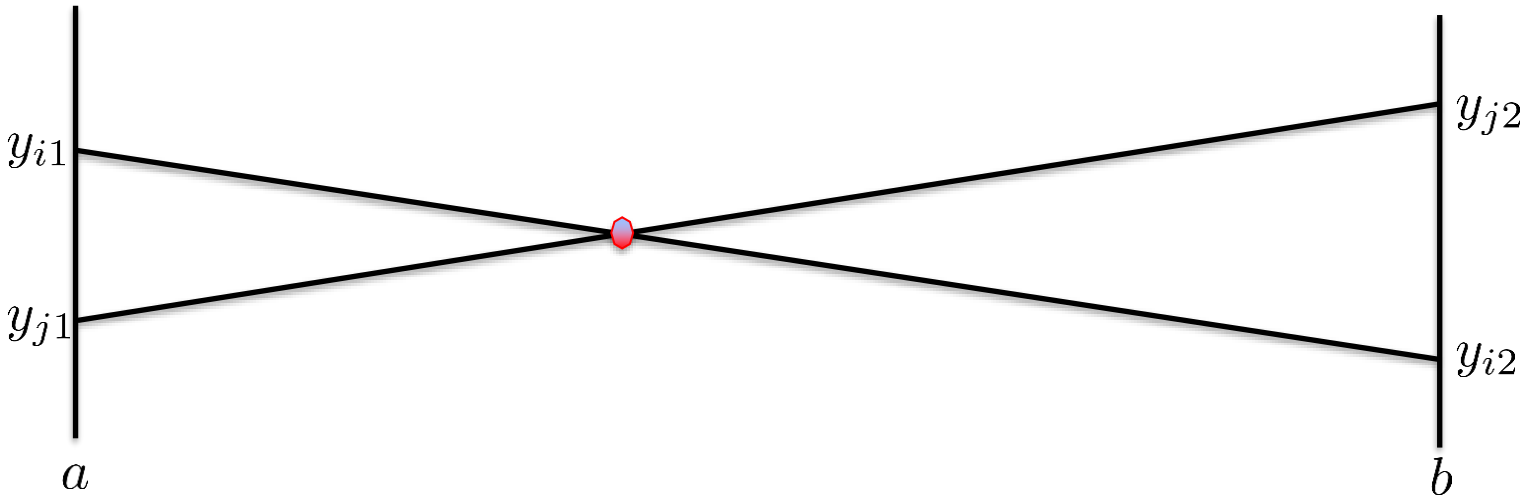


Case study II: Counting intersections

Problem: Given n distinct lines in the plane, none of which are vertical and two vertical lines $x = a$ and $x = b$, find the number of intersections. We assume that each line i is described by its endpoints (a, y_{i1}) and (b, y_{i2}).

Question: When two lines i, j intersect?

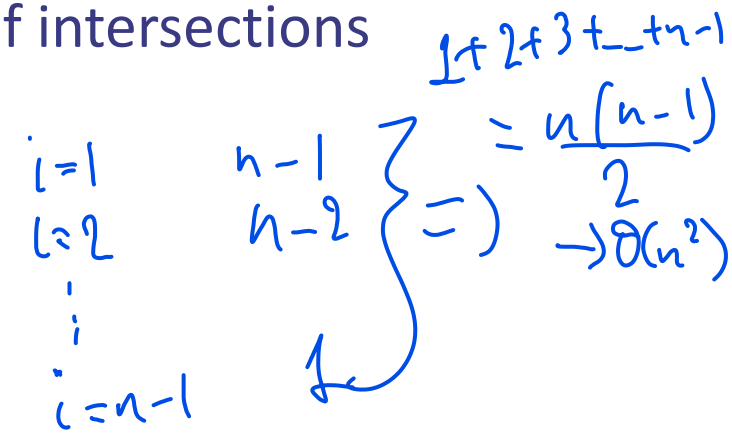
If $\boldsymbol{y_{i1} > y_{j1}}$ **then** $\boldsymbol{y_{i2} < y_{j2}}$ or If $\boldsymbol{y_{i1} < y_{j1}}$ **then** $\boldsymbol{y_{i2} > y_{j2}}$



Case study II: Counting intersections

For all pairs i, j with $i < j$, count number of intersections

Pseudocode:



counter $\leftarrow 0$
For $i = 1$ to n **do**
  For $j = i + 1$ to n **do**
    If $(y_{i1} > y_{j1}$ and $y_{i2} < y_{j2})$ or $(y_{i1} < y_{j1}$ and $y_{i2} > y_{j2})$ **then**
      counter $\leftarrow$ counter $+ 1$
return counter

Case study II: Counting intersections

For all pairs i, j with $i < j$, count number of intersections

Pseudocode:

```
counter ← 0
For i = 1 to n do
    For j = i + 1 to n do
        If (y_i1 > y_j1 and y_i2 < y_j2) or (y_i1 < y_j1 and y_i2 > y_j2) then
            counter ← counter + 1
return counter
```

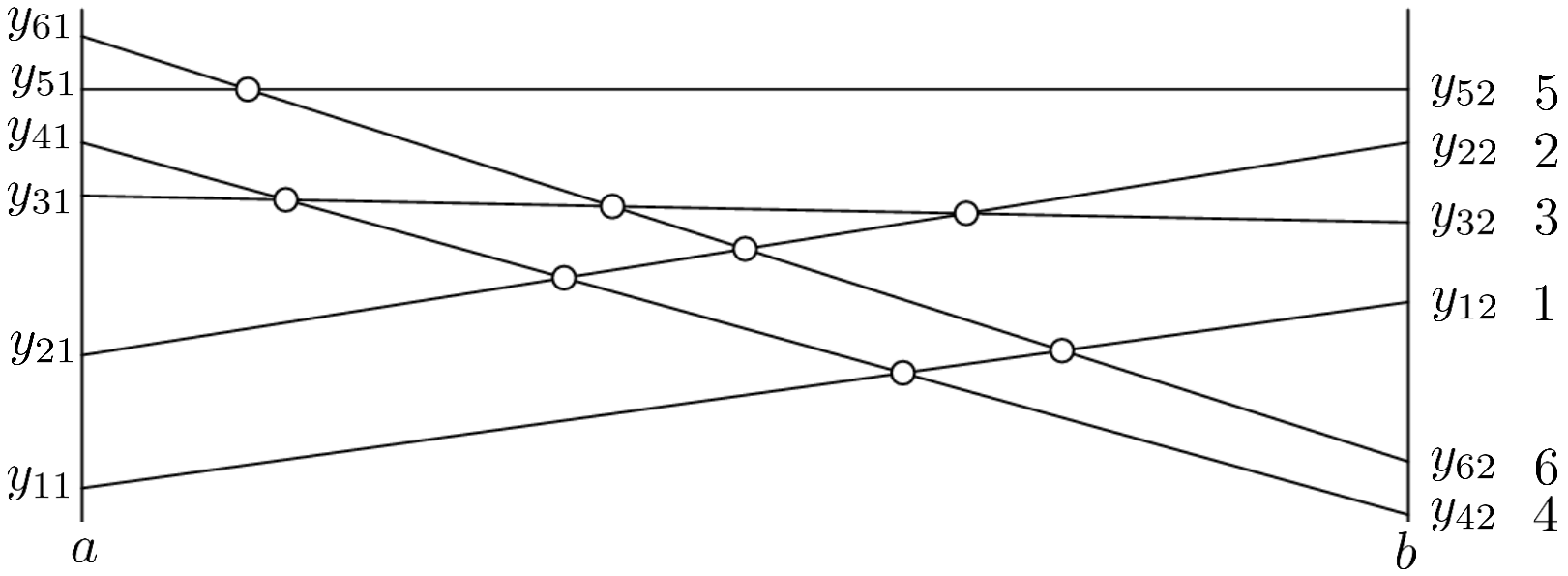
Running time $\Theta(n^2)$

Can we do better?

Case study II: Counting intersections

Idea: Let's sort the lines with respect to y on a. Check the inverse permutation of the indices of the lines on b.

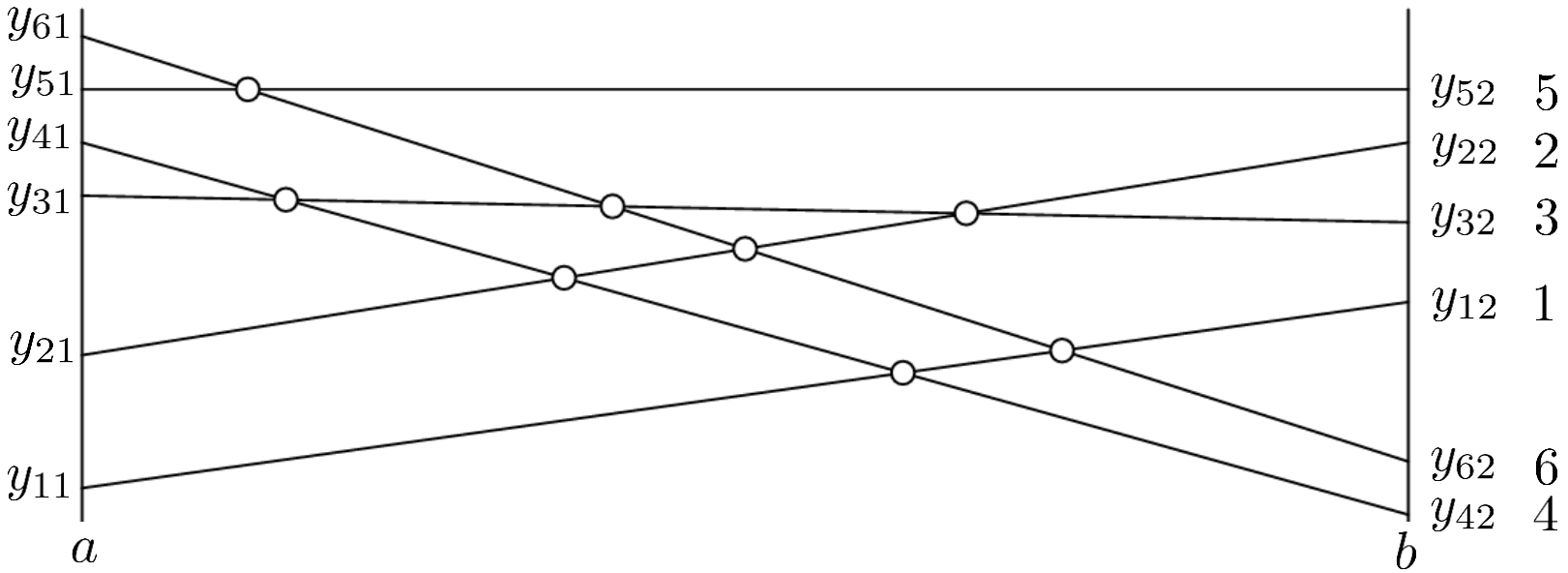
Example: 4, 6, 1, 3, 2, 5



Case study II: Counting intersections

Idea: Let's sort the lines with respect to y on a. Check the inverse permutation of the indices of the lines on b.

Example: 4, 6, 1, 3, 2, 5

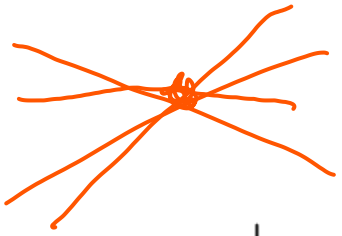


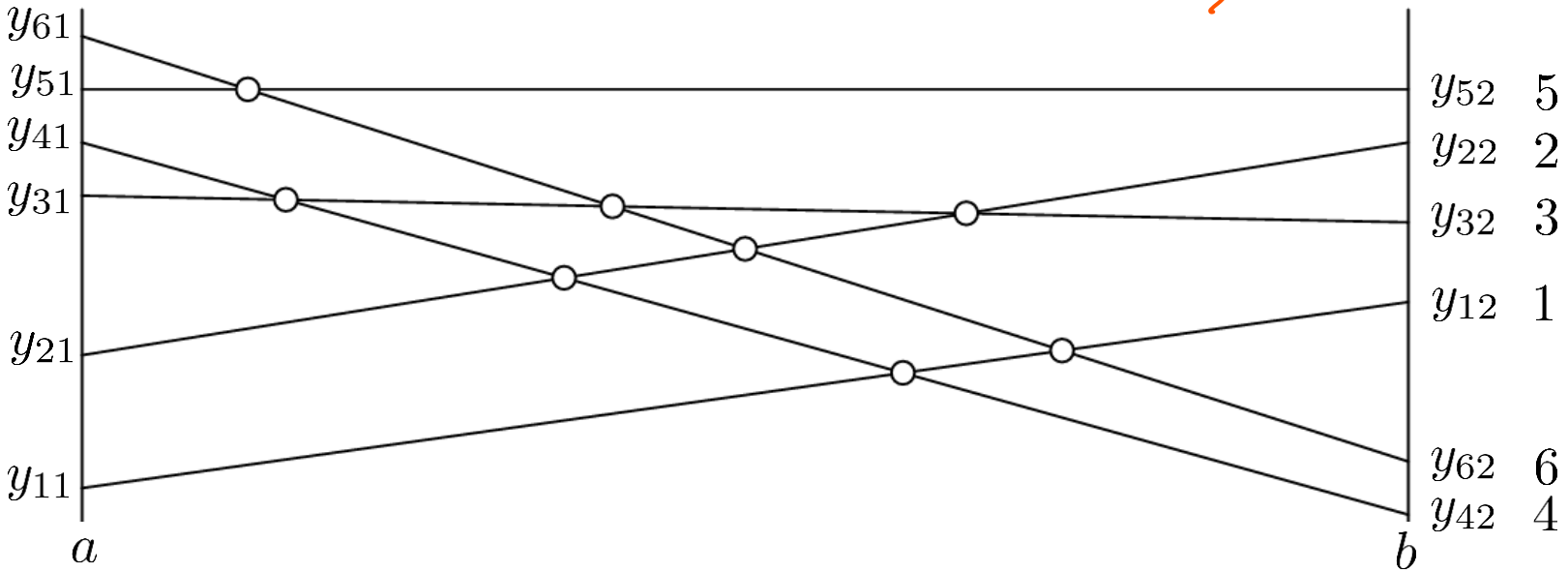
Key observation: Number of inversions is equal to number of intersections. In example (4, 1), (4,3), (4,2), (6,1), (6,3), (6,2), (6,5), (3,2)

Case study II: Counting intersections

Idea: Let's sort the lines with respect to y on a. Check the inverse permutation of the indices of the lines on b.

Example: 4, 6, 1, 3, 2, 5





Solution: Sort the lines with respect to y on a. Run modified mergesort to find number of inversions. Running time $\Theta(n \log n)$.

Case study III: Maximum subarray

Problem (Leetcode, question in interviews): Given an array A of n numbers (positive and negative), find the subarray with the maximum sum.

Example: $A = [-2, -5, 6, -2, -3, 1, 5, -6]$

Case study III: Maximum subarray

Problem (Leetcode, question in interviews): Given an array A of n numbers (positive and negative), find the subarray with the maximum sum.

Example: $A = [-2, -5, 6, -2, -3, 1, 5, -6]$

Solution of example:

$[-2, -5, \mathbf{6, -2, -3, 1, 5}, -6]$ with sum 7.

Case study III: Maximum subarray

For all i, j with $i \leq j$, compute $A_i + A_{i+1} + \cdots + A_j$. Keep the maximum from all sums. Total number of computations is…

Case study III: Maximum subarray

For all i, j with $i \leq j$, compute $A_i + A_{i+1} + \cdots + A_j$. Keep the maximum from all sums. Total number of computations is…

Pseudocode:

$\max \leftarrow 0$
For $i = 1$ to n **do**
    For $j = i$ to n **do**
        $\text{sum} \leftarrow 0$
        For $k = i$ to j **do**
            $\text{sum} = \text{sum} + A[k]$
        If $\text{sum} > \max$ **then**
            $\max \leftarrow \text{sum}$
return $\max$

Case study III: Maximum subarray

For all i, j with $i < j$, compute $A_i + A_{i+1} + \cdots + A_j$. Keep the maximum from all sums. Total number of computations is

$$\sum_{i=1}^{n} \sum_{j=i}^{n} (j - i + 1) \text{ which is } \Theta(n^3)$$

Pseudocode:

```
max ← 0
For i = 1 to n do
    For j = i to n do
        sum ← 0
        For k = i to j do
            sum = sum + A[k]
        If sum > max then
            max ← sum
return max
```

Can we do better?

Case study III: Maximum subarray

Idea 1: Do first preprocessing. Compute partial sums $S_i = A_1 + \cdots + A_i$ for every i. Running time $\Theta(n)$.

Observe that $A_i + A_{i+1} + \cdots + A_j = S_j - S_{i-1}$

$S_j = A_1 + \cdots + A_{i-1} + A_i + A_{i+1} + \cdots + A_j$

$S_{i-1} = A_1 + \cdots + A_{i-1}$

$S_j - S_{i-1} =$

Case study III: Maximum subarray

Idea 1: Do first preprocessing. Compute partial sums

$S_i = A_1 + \cdots + A_i$ for every i. Running time $\Theta(n)$.

Observe that $A_i + A_{i+1} + \cdots + A_j = S_j - S_{i-1}$

Then for all i, j with $i \leq j$, compute the maximum among $S_j - S_{i-1}$.

Case study III: Maximum subarray

Idea 1: Do first preprocessing. Compute partial sums

$S_i = A_1 + \cdots + A_i$ for every i. Running time $\Theta(n)$.

Observe that $A_i + A_{i+1} + \cdots + A_j = S_j - S_{i-1}$

Then for all i, j with $i \leq j$, compute the maximum among $S_j - S_{i-1}$.

$\max \leftarrow 0$
$S[0] \leftarrow 0$
For $i = 1$ to n **do**
  $S[i] \leftarrow S[i-1] + A[i]$
For $i = 1$ to n **do**
  For $j = i$ to n **do**
    If $S[j] - S[i-1] > \max$ **then**
      $\max \leftarrow S[j] - S[i-1]$
return max

$[-2, -5, 6, -2, -3, 1, 5, -6]$
S = [0, -2, -7, -1, -3, -6, -5, 0, -6]
-5+6-2-3+1 = S[6] – S[1] = -3

Case study III: Maximum subarray

Idea 1: Do first preprocessing. Compute partial sums $S_i = A_1 + \cdots + A_i$ for every i. Running time $\Theta(n)$.

Observe that $A_i + A_{i+1} + \cdots + A_j = S_j - S_{i-1}$

Then for all i, j with $i < j$, compute the maximum among $S_j - S_{i-1}$.

$\max \leftarrow 0$
$S[0] \leftarrow 0$
For $i = 1$ to n **do**
  $S[i] \leftarrow S[i-1] + A[i]$
For $i = 1$ to n **do**
  For $j = i$ to n **do**
    If $S[j] - S[i-1] > \max$ **then**
      $\max \leftarrow S[j] - S[i-1]$
return max

Running time $\Theta(n^2)$

Can we do better?

Case study III: Maximum subarray

Idea 2: Divide and conquer

$$[-2, -5, 6, -2, -3, 1, 5, -6]$$

Find max in left half (e.g., green), find max in right half (e.g., black) and combine/merge. HOW?

Observe left part has maximum 6 and right part also 6. The solution is 7 though.

Case study III: Maximum subarray

Idea 2: Divide and conquer

$$[-2, -5, 6, -2, -3, 1, 5, -6]$$

Find max in left half (e.g., green), find max in right half (e.g., black) and combine/merge. HOW?

Observe left part has maximum 6 and right part also 6. The solution is 7 though.

Key idea: The solution is either on **left** part, or **right** part or **crosses the midpoint (has at least one number in both parts).**

Case study III: Maximum subarray

Idea 2: Divide and conquer

$$[-2, -5, 6, -2, -3, 1, 5, -6]$$

Question: How to get the maximum subarray that crosses the midpoint?

Case study III: Maximum subarray

Idea 2: Divide and conquer

$$[-2, -5, 6, -2, -3, 1, 5, -6]$$

Question: How to get the maximum subarray that crosses the midpoint?

a) Find the maximum starting from mid and going left.

b) Find the maximum starting from mid+1 and going right.

Add them up. This can happen in $\Theta(n)$ time using partial sums $\boldsymbol{S}$.

In example above a) is 4 and b) is 3 for a total of 7.

$T(n) = 2T(n/2) + \Theta(n)$ $\Theta(n)$

Case study III: Maximum subarray

Pseudocode:

$S[0] \leftarrow 0$ **For** $i = 1$ to n **do**
$S[i] \leftarrow S[i-1] + A[i]$

$S_i = A_1 + \dots + A_i$ preprocessing for partial sums.

Maxsum$(A[1:n])$

$\Theta(1)$ **If** $n == 1$ **return** $\max(A[1], 0)$
$T(n/2)$ **maxL** $\leftarrow$Maxsum$(A[1:n/2])$
$T(n/2)$ **maxR** $\leftarrow$Maxsum$(A[n/2+1:n])$
} Divide & Conquer

$\Theta(1)$ **max1** $\leftarrow 0$
max2 $\leftarrow 0$
$\Theta(n/2)$ **For** $i = n/2$ downto 1 **do**
If max1 $< S[n/2] - S[i-1]$ **then max1** $\leftarrow S[n/2] - S[i-1]$
$\Theta(n/2)$ **For** $i = n/2 + 1$ to n **do**
If max2 $< S[i] - S[n/2]$ **then max2** $\leftarrow S[i] - S[n/2]$
} compute max subarray that crosses midpoint & combine sol'n

$\Theta(1)$ **return** maximum of maxL, maxR and max1 + max2

Case study III: Maximum subarray

Pseudocode:

$S[0] \leftarrow 0$ **For** $i = 1$ to n **do**
$S[i] \leftarrow S[i-1] + A[i]$

$[-2, -5, 6, -2, -3, 1, 5, -6]$
maxL = 6, maxR = 6, max1+max2 = 4+3 = 7

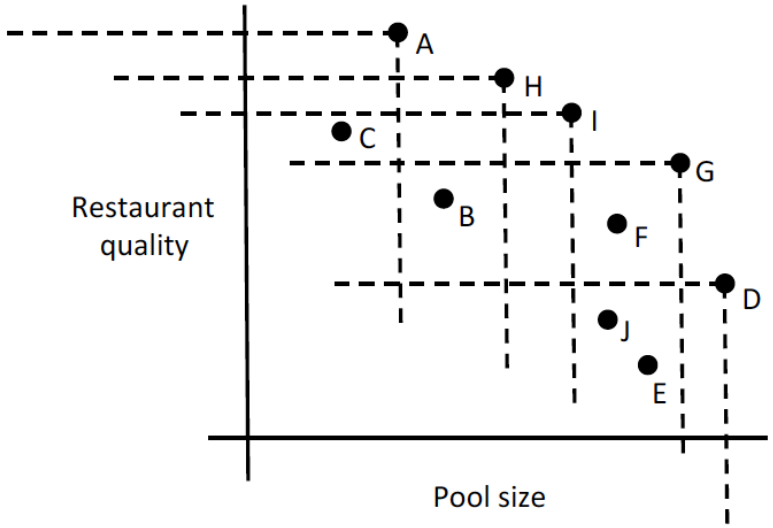
$\text{Maxsum}(A[1:n])$

If $n == 1$ **return** $\max(A[1], 0)$
maxL $\leftarrow$Maxsum$(A[1:n/2])$
maxR $\leftarrow$Maxsum$(A[n/2+1:n])$
max1 $\leftarrow 0$
max2 $\leftarrow 0$
For $i = n/2$ downto 1 **do**
  If max1 $< S[n/2] - S[i-1]$ **then max1** $\leftarrow S[n/2] - S[i-1]$
For $i = n/2 + 1$ to n **do**
  If max2 $< S[i] - S[n/2]$ **then max2** $\leftarrow S[i] - S[n/2]$
return maximum of maxL, maxR and max1 + max2

Case study IV: Maxima Set

Problem: We are given n points $(x_1, y_1), \ldots, (x_n, y_n)$ on the plane. A point (x_i, y_i) is called a maximum point if there is no other point (x_j, y_j) that $x_i \leq x_j$ and $y_i \leq y_j$.

Example: x captures pool size and y restaurant quality. 10 hotels



Case study IV: Maxima Set

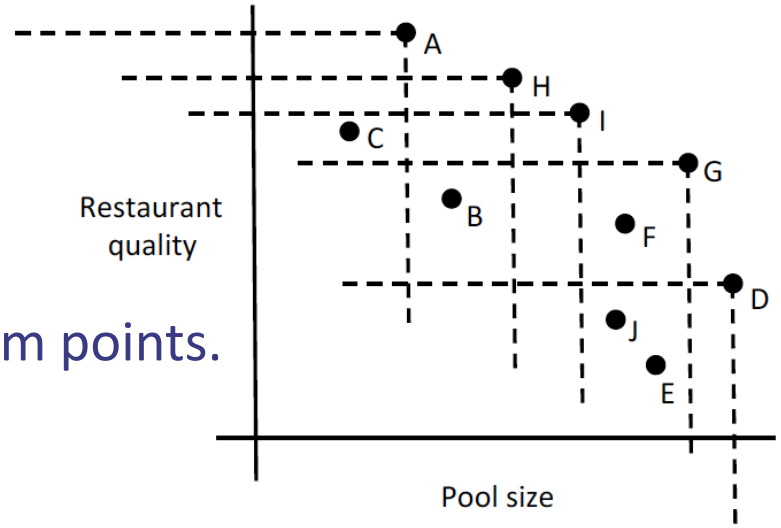
Problem: We are given n points $(x_1, y_1), \ldots, (x_n, y_n)$ on the plane. A point (x_i, y_i) is called a maximum point if there is no other point (x_j, y_j) that $x_i \leq x_j$ and $y_i \leq y_j$.

Example: x captures pool size and y restaurant quality. 10 hotels

Explanation:

A, H, G, D are maximum points.

C, B, F, J, E are not.



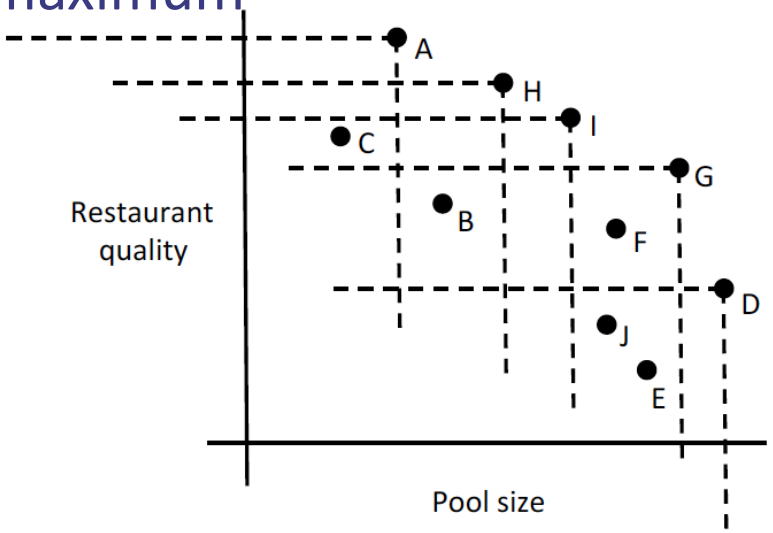
Case study IV: Maxima Set

Problem: We are given n points $(x_1, y_1), \ldots, (x_n, y_n)$ on the plane. A point (x_i, y_i) is called a maximum point if there is no other point (x_j, y_j) that $x_i \leq x_j$ and $y_i \leq y_j$.

Obvious approach:

For every point (x_i, y_i), check if it is maximum

To check if it is maximum, you check the condition with all other points.



Case study IV: Maxima Set

Problem: We are given n points $(x_1, y_1), \ldots, (x_n, y_n)$ on the plane. A point (x_i, y_i) is called a maximum point if there is no other point (x_j, y_j) that $x_i \leq x_j$ and $y_i \leq y_j$.

Pseudocode:

```
counter ← 0
For i = 1 to n do
flag ← 1
    For j = i + 1 to n do
        If (x_j > x_i and y_j > y_i) then flag ← 0
counter ← counter + flag
return counter
```

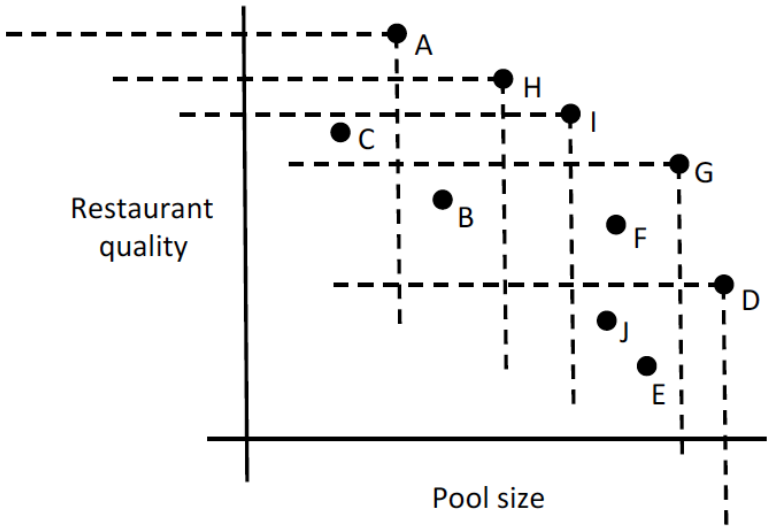
Running time $\Theta(n^2)$

Can we do better?

Case study IV: Maxima Set

Problem: We are given n points $(x_1, y_1), \ldots, (x_n, y_n)$ on the plane. A point (x_i, y_i) is called a maximum point if there is no other point (x_j, y_j) that $x_i \leq x_j$ and $y_i \leq y_j$.

Idea: Divide and conquer. Divide step and Combine step is challenging.



Case study IV: Maxima Set

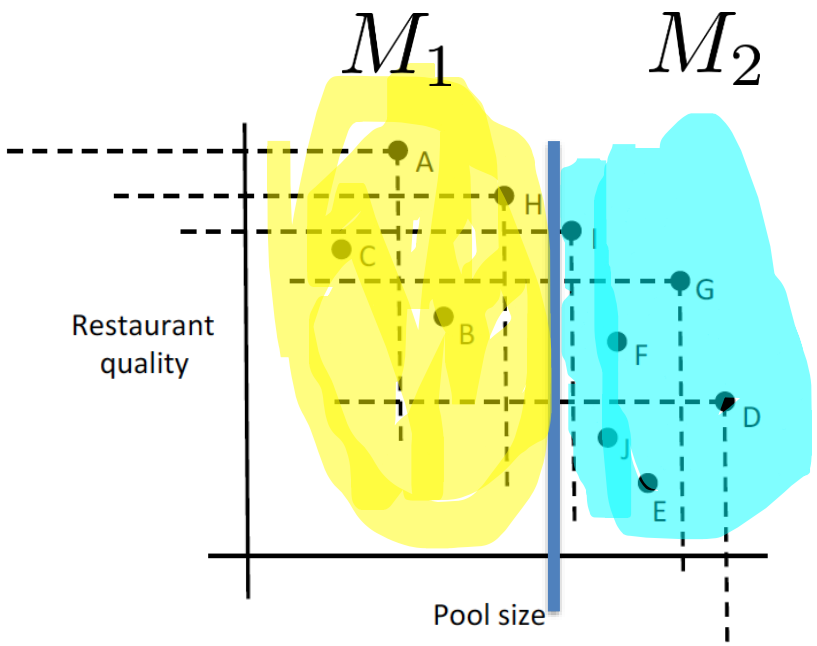
Divide step: It should split the points in two parts of equal size.

How?

Case study IV: Maxima Set

Divide step: It should split the points in two parts of equal size.

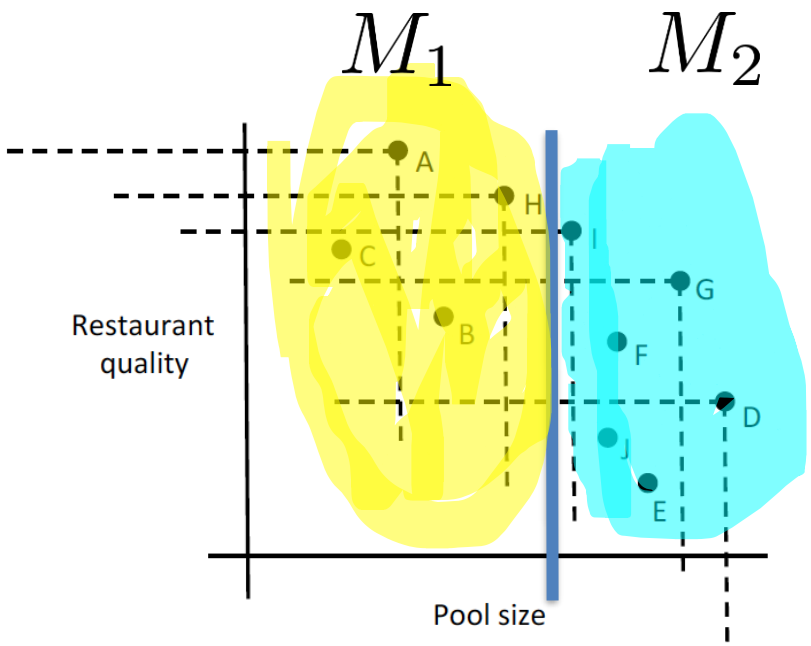
How? Choose the middle (median) point with respect to x coordinates.



Case study IV: Maxima Set

Divide step: It should split the points in two parts of equal size.

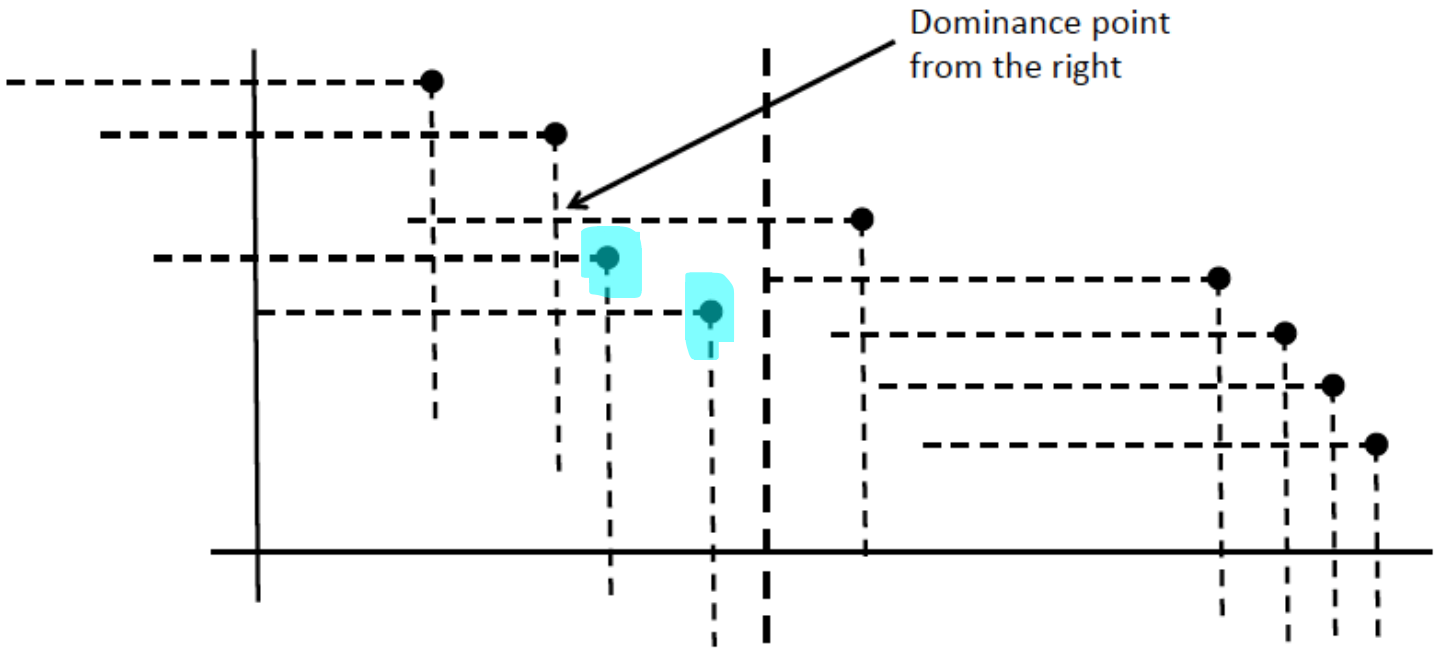
How? Choose the middle (median) point with respect to x coordinates.



Combine step: Return $M_1 \cup M_2$?

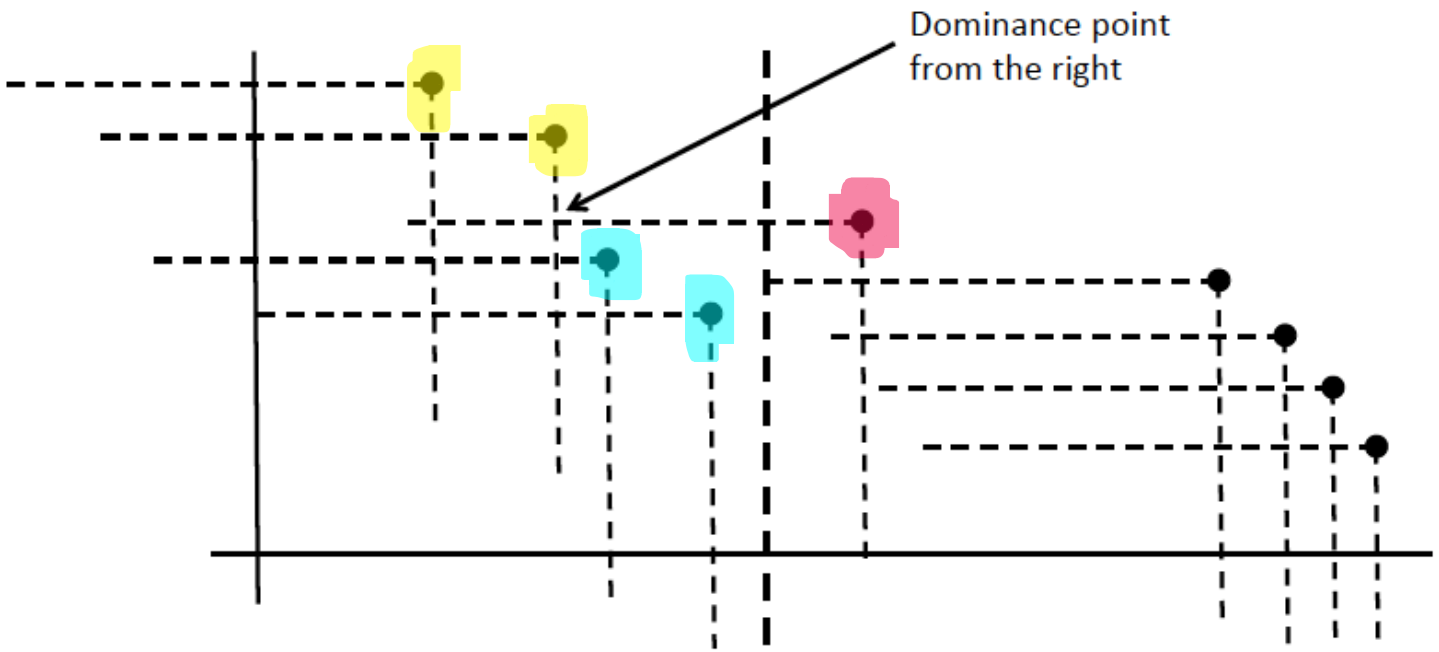
Case study IV: Maxima Set

Combine step: Return $M_1 \cup M_2$? Wrong: blue points below of M_1 are not part of the solution



Case study IV: Maxima Set

Combine step idea: M_2 points should part of the solution. From M_1, the points that are maximum should not be dominated by smallest with respect to x coordinates in M_2



Case study IV: Maxima Set

Pseudocode:

MaximaSet(S,n):
    if $n = 1$ **then**
        return S
    Let p be the median point in S, by x -coordinates
    Let L be the set of points less than p in S by x -coordinates
    Let G be the set of points greater than or equal to p in S by x -coordinates
    $M_1 \leftarrow$ MaximaSet(L)
    $M_2 \leftarrow$ MaximaSet(G)
    Let q be the smallest point in M_2
    for each point, r, in M_1 **do** by x -coordinates
        if $x(r) \leq x(q)$ **and** $y(r) \leq y(q)$ **then**
            Remove r from M_1
    return $M_1 \cup M_2$

Case study IV: Maxima Set

Pseudocode:

MaximaSet(S,n):
  if $n = 1$ **then**
    return S
  Let p be the median point in S, by x -coordinates
  Let L be the set of points less than p in S by x -coordinates
  Let G be the set of points greater than or equal to p in S by x -coordinates
  $M_1 \leftarrow$ MaximaSet(L)
  $M_2 \leftarrow$ MaximaSet(G)
  Let q be the smallest point in M_2
  for each point, r, in M_1 **do** by x -coordinates
    if $x(r) \leq x(q)$ **and** $y(r) \leq y(q)$ **then**
      Remove r from M_1
  return $M_1 \cup M_2$

$$\text{Running time is } T(n) = 2T(n/2) + T_{\text{media}}(n) + T_{\min}(n) + \Theta(n)$$
$$= 2T(n/2) + T_{\text{media}}(n) + \Theta(n)$$

Case study IV: Maxima Set

Pseudocode:

M

Next week we will see how to find the median in $\Theta(n)$ time!
This fact will yield Θ(nlog n) for Maxima Set.

rdinates
in S by x -coordinates

$M_2 \leftarrow$ MaximaSet(G)
Let q be the smallest point in M_2
for each point, r, in M_1 **do** by x -coordinates
  if $x(r) \leq x(q)$ **and** $y(r) \leq y(q)$ **then**
    Remove r from M_1
return $M_1 \cup M_2$

$$\text{Running time is } T(n) = 2T(n/2) + T_{\text{media}}(n) + T_{\min}(n) + \Theta(n)$$
$$= 2T(n/2) + T_{\text{media}}(n) + \Theta(n)$$