# L01 Introduction

CS 295 Introduction to Algorithmic Game Theory

Ioannis Panageas

# Course material

We will use canvas and material will also be posted on
https://panageas.github.io/agt2021/

Recommended Textbooks

- Nisan/Roughgarden/Tardos/Vazirani (eds), Algorithmic Game Theory (online).
- Tim Roughgarden notes (online).

Many lectures will not be part of the above!

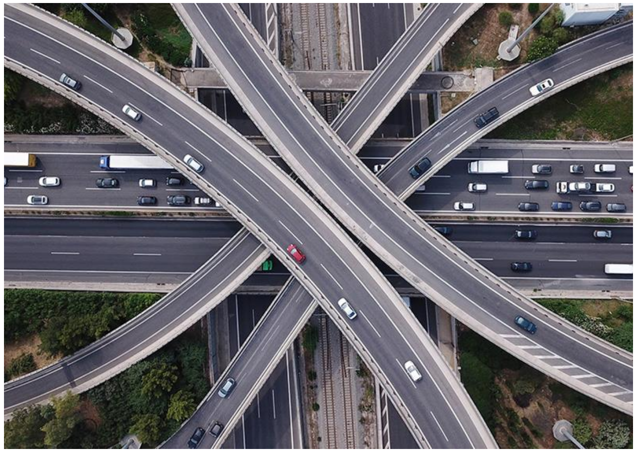
# Grading

- Participation : 5%
- Homework: 30%
  - There will be given 2 Homeworks to solve (**Latex!**).
- Scribing lecture notes: 30%
  - **Latex template**, Group of 2-3. Deadline 3 weeks after the lecture.
- Research Project/Present paper : 35%
  - Group of ~3. Report Deadline on 7th of December.
  - Presentation last week of classes.

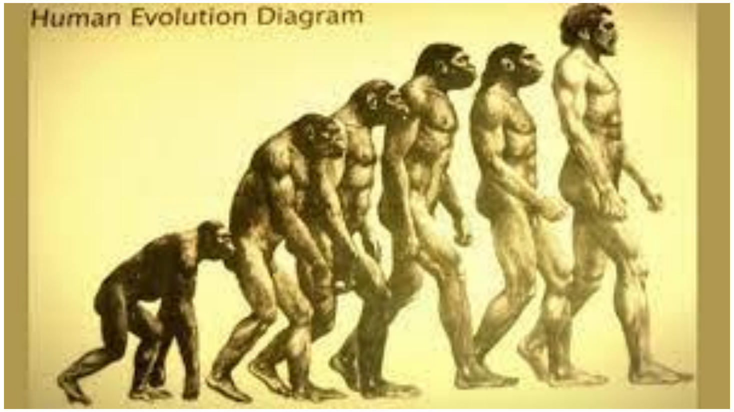
# What is Game Theory?

**Markets - Auctions**


**Routing**



**Evolution**




**Elections**

# What is Game Theory?

***Games*** are thought experiments helping us to ***predict rational behavior*** in ***situations of conflict.***

1. ***Conflict:*** Everybody's actions affect others.
2. ***Rational Behavior:*** The players want to maximize their own expected utility.
3. ***Predict:*** We want to know what happens. Via solution concepts.

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# Example: Prisoner's Dilemma

Simultaneously, the police offer each prisoner a bargain:

- If A and B both confess, each of them serves 2 years in prison.
- If A confesses but B denies, A will be set free and B will serve 3 years in prison (and vice versa).
- If both A and B deny the crime, they will both serve 1 year in prison.

| | Deny | Confess |
|---|---|---|
| Deny | **1, 1** | **3, 0** |
| Confess | **0, 3** | **2, 2** |

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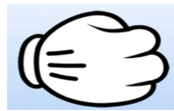
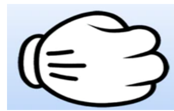
**How A and B will play?**

# Example: Rock-Paper-Scissors

| |  |  |  |
|---|---|---|---|
|  | 0, 0 | -1, 1 | 1,-1 |
|  | 1,-1 | 0, 0 | -1, 1 |
|  | -1, 1 | 1, -1 | 0, 0 |

# Example: Rock-Paper-Scissors

| |  |  |  |
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**No dominant strategy equilibrium!**

Concept: **Nash Equilibrium**

A pair of strategies (deterministic or randomized) such that the strategy of the row player is at least as good as any other strategy of her given the strategy of the column player (and vice versa).

# Bimatrix Games

- 2 players: Row and Column
- $n, m$ strategies available
- Payoff matrices $R, C$ of size $n \times m$.

$$\mathcal{G} = (R_{n\times m}, C_{n\times m})$$

payoff of the column player
for playing $j$
when row player plays $i$.

$$R_{ij}, C_{ij}$$

payoff of the row player for playing $i$
when column player plays $j$.

# Bimatrix Games

Column player
chooses $y \in \Delta_m$

Row player
chooses $x \in \Delta_n$

$$R_{ij}, C_{ij}$$

Row gets $x^\top R y$.

Column gets $x^\top C y$.

# Bimatrix Games

Column player
chooses $y \in \Delta_m$

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$$R_{ij}, C_{ij}$$

Row gets $x^\top R y$.
Column gets $x^\top C y$.

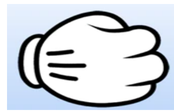
**Definition** (Nash Equilibrium). $(x^*, y^*)$ *is a Nash Equilibrium iff for all possible randomized strategies* $x'$ *of row player holds*

$$x^{*\top} R y^* \geq x'^\top R y^*$$

and for all possible randomized strategies $y'$ of column player holds

$$x^{*\top} C y^* \geq x^{*\top} C y'.$$

# Example: Rock-Paper-Scissors

| |  |  |  |
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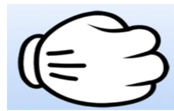
|  | 1,-1 | 0, 0 | -1, 1 |
|  | -1, 1 | 1, -1 | 0, 0 |

*The unique Nash Equilibrium is* $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

**Remark**:

Contrary to Prisoner's Dilemma, in RPS ***randomization is necessary*** for Nash equilibrium to exist!

# Example: Rock-Paper-Scissors

| |  |  |  |
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|  | -1, 1 | 1, -1 | 0, 0 |

***Nice solution concept but does it always exist?***

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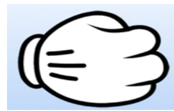
# Example: Rock-Paper-Scissors

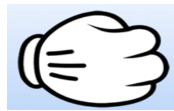
| |  |  |  |
|---|---|---|---|
|  | 0, 0 | -1, 1 | 1,-1 |
|  | 1,-1 | 0, 0 | -1, 1 |
|  | [illegible] | [illegible] | 0, 0 |

**von Neumann ’28:**
*For **two-player zero-sum games**, i.e., $R + C = 0$, it **always exists**!*

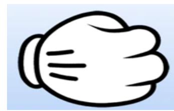
**Remark**:

Contrary to Prisoner’s Dilemma, in RPS ***randomization is necessary*** for Nash equilibrium to exist!

# Example: Modified Rock-Paper-Scissors

| |  |  |  |
|---|---|---|---|
|  | 0, 0 | -1, 1 | 2,-1 |
|  | 1,-1 | 0, 0 | -1, 1 |
|  | -2, 1 | 1, -1 | 0, 0 |

# Example: Modified Rock-Paper-Scissors

| |  |  |  |
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|  | 0, 0 | -1, 1 | 2,-1 |
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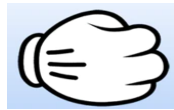
|  | -2, 1 | 1, -1 | 0, 0 |

**Not zero sum anymore!**

# Example: Modified Rock-Paper-Scissors

| | 25%  | 50%  | 25%  |
|---|---|---|---|
| 33.3%  | 0, 0 | -1, 1 | 2,-1 |
| 33.3%  | 1,-1 | 0, 0 | -1, 1 |
| 33.3%  | -2, 1 | 1, -1 | 0, 0 |

**Not zero sum anymore!**

John Nash '51:

There always **exists** a Nash equilibrium (finite games)!

# Cool but **Algorithmic**?

Question: Can we predict what will happen in a **large** system?

- ***Computing Nash Equilibrium***: Design fast Algorithms to compute Nash Equilibrium!
- ***Mechanism Design***: Design a system that will be used by users to optimize our objectives!

# Cool but **Algorithmic**?

Question: Can we predict what will happen in a **large** system?
Game theory: Yes, via solution concept (system will reach equilibrium).

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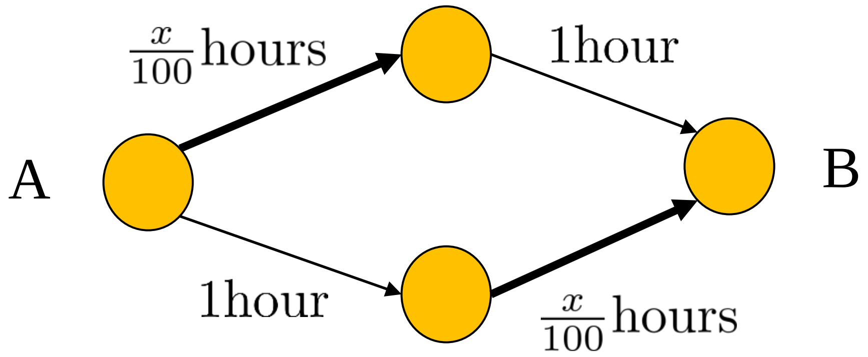
# Cool but **Algorithmic**?

Question: Can we predict what will happen in a **large** system?
Game theory: Yes, via solution concept (system will reach equilibrium).
Question: How to **compute efficiently** an equilibrium?

- ***Computing Nash Equilibrium***: Design fast Algorithms to compute Nash Equilibrium!
- ***Mechanism Design***: Design a system that will be used by users to optimize our objectives!

# Price of Anarchy

Suppose 100 drivers commute from A to B.
Drivers want to minimize the time.



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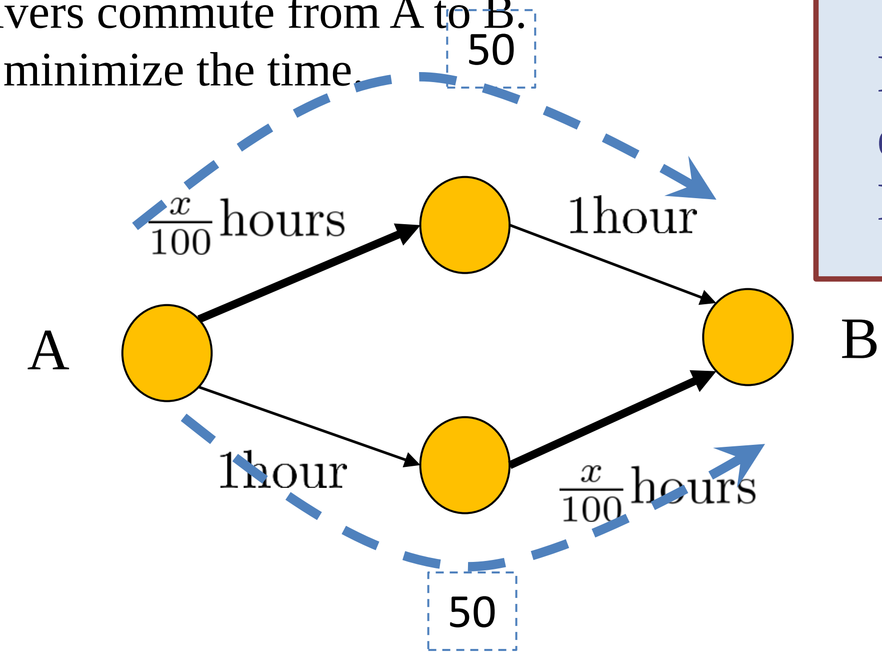


Delay is 1.5 hours for everybody at the unique Nash equilibrium.

# Price of Anarchy

Suppose 100 drivers commute from A to B.
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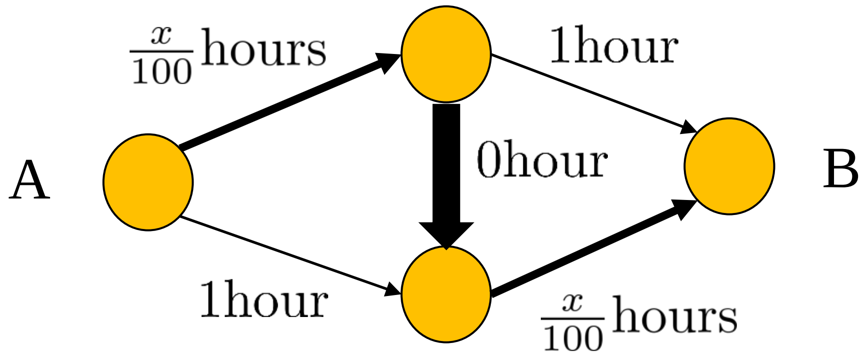
Question: What if we add a new link?



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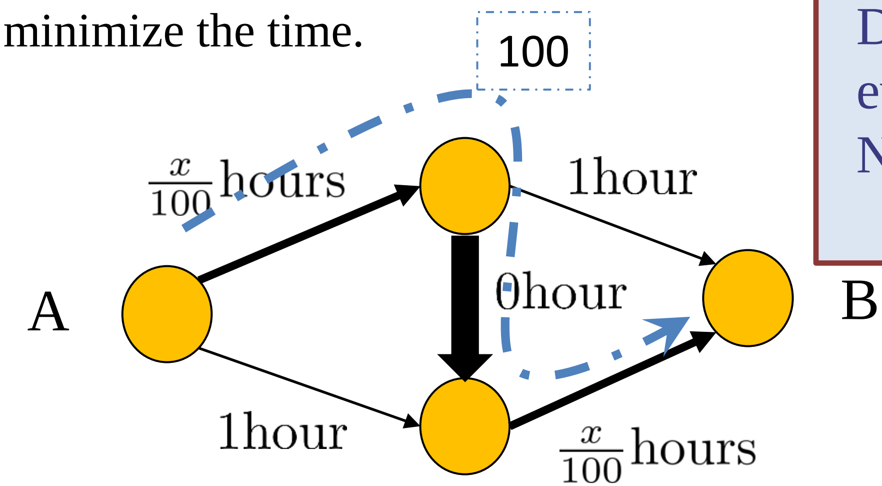


Delay is now 2 hours for everybody at the unique Nash equilibrium.

Braess's paradox

Adding a fast link is not always a good idea!

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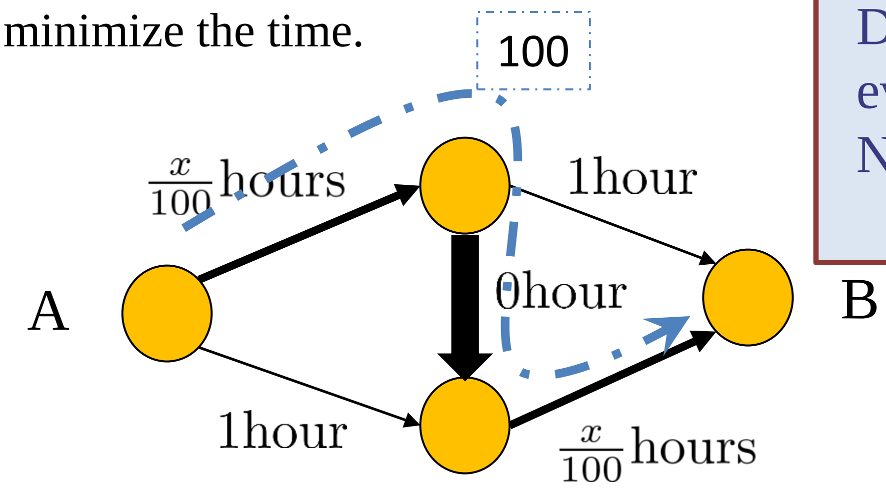


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$$\text{PoA} = \frac{\text{performance of worst case NE}}{\text{optimal performance if agents do not decide on their own}}$$

Price of Anarchy (Koutsoupias, Papadimitriou 99').

**4/3!!**

# Auctions

- Auctioneer has one item for sale.
- $n$ bidders are interested in the item.
- Bidder $i$ has valuation $v_i$ for the item (unknown to Auctioneer).
- Each bidder $i$ places a bid $b_i$, and based on $b_1, \ldots, b_n$ auctioneer decides who gets the item and how much to pay.
- If bidder $i$ gets the item and pays price $p$, her utility is $v_i - p$ otherwise 0.

Goal: Auctioneer wants to maximize her revenue! What is the correct pricing? Who will get the item?